Appendix II to the paper published in transactions of Scientific Journal Physics, 2011 355(16) , pages 133-138.

How approximate solution of an initial value problem of finding amplitude of every field component in light waves in self focused kerr media as function of dielectric function is obtained is demonstrated in the second approximation.

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Seeks to delermine square amplitude of y component of magnetic field using equations formulated by us. After second approximation the expresions for square of amplitude of y component of magnetic field and Dd-package will look as

h^2=h0 x+h1 x^2+h2 x^3 and Dd=Dd0+Dd1 x+Dd2 x^2 (see formulas :(18a) - (18c) and (17) in the paper)

where h2 and Dd2 shoud be determined, in the second approximation, in the term of initial value, h0. we have extensively used several commands of symbolic calculus of mathematic 6 and our presentation below was given in the form of a notebook of mathematic 6

Notation: $b0=\beta^2$, **ep=dielectric constant.**

```
h0 =.; h1 =.; h2 =.; x1 =.; x2 =.; x3 =.; b11 =.; b21 =.; b12 =.; b22 =.;
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```
Clear[Dd0, Dd1, Dd2]
```

Quantities h1 and Dd0, Dd1 have been calculated, from previous approximations, in the term of initial value, h0.

$$h1 = \frac{h0}{2\,ep} + \frac{2\,b0\,h0^2}{ep^3};\ Dd0 = -ep\left(\frac{1}{h0} + \frac{ep - 2\,b0}{ep\,^\wedge 2}\right);\ Dd1 = 0;$$

{Aa0, Aa1, Aa2} is coefficientList in power series of square of amplitude of x - component of the electric field, A^2. To express elements of this list in terms of h0, h1, h2 we use equation :

$$\frac{d}{d\epsilon} A\,^\wedge 2 = \frac{(b0 - \epsilon)}{\epsilon\,^\wedge 2}\ \frac{d}{d\epsilon} h^2$$

$$b1 = b0 - ep;$$

$$x1 = \frac{(b1 - x)\,(h0 + 2\,h1\,x + 3\,h2\,x\,^\wedge 2)}{(ep + x)\,^\wedge 2};$$

```
x2 = Normal[Series[x1, {x, 0, 2}]];
```

```
Aa = CoefficientList[x2, x] {1, 1/2, 1/3};
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```
{Aa0, Aa1, Aa2} = Collect[Aa, {h0, h3}];
```

To decompose Dd2 we need following list

```
{Aa0, Aa1, Aa2} / Aa0;
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{Aa0a, Aa1a, Aa2a} = Normal[Series[%, {h0, 0, 12}]]

$$\left\{1, -\frac{b0}{2\,(b0-ep)\,ep}+\frac{2\,b0\,h0}{ep^3}, \frac{b0}{3\,(b0-ep)\,ep^2}-\frac{4\,(2\,b0^2-b0\,ep)\,h0}{3\,(b0-ep)\,ep^4}+\frac{h2}{h0}\right\}$$

Aa2a

$$\frac{b0}{3\,(b0-ep)\,ep^2}-\frac{4\,(2\,b0^2-b0\,ep)\,h0}{3\,(b0-ep)\,ep^4}+\frac{h2}{h0}$$

Discarding terms with factor h2 from this expression.

h2 = 0; Aa2a; h2 =.;

e_y^2 = e0 x + e1 x^2 + e2 x^3. To get {e0, e1, e2} – list, in terms of h0, h1, h2, we use

equation: $e_y^2 = \epsilon - ep - A\hat{}2 - \frac{b0}{\epsilon\hat{}2}\,h\hat{}2.$

$$x3 = 1 - \text{Normal}\left[\text{Series}\left[\frac{b0}{(ep+x)\hat{}2}\,(h0 + h1\,x + h2\,x\hat{}2), \{x, 0, 2\}\right]\right];$$

CoefficientList[%, x] – Aa;

{e0, e1, e2} = Collect[%, {h0, h2}]

$$\left\{1+\left(-\frac{2\,b0}{ep^2}+\frac{1}{ep}\right)h0, \frac{2\,b0\,h0}{ep^3}+\left(-\frac{4\,b0^2}{ep^5}+\frac{2\,b0}{ep^4}\right)h0^2, -\frac{7\,b0\,h0}{3\,ep^4}+\left(\frac{20\,b0^2}{3\,ep^6}-\frac{4\,b0}{3\,ep^5}\right)h0^2+\left(-\frac{2\,b0}{ep^2}+\frac{1}{ep}\right)h2\right\}$$

$$\left\{1+\left(-\frac{2\,b0}{ep^2}+\frac{1}{ep}\right)h0, \frac{2\,b0\,h0}{ep^3}+\left(-\frac{4\,b0^2}{ep^5}+\frac{2\,b0}{ep^4}\right)h0^2, -\frac{7\,b0\,h0}{3\,ep^4}+\left(\frac{20\,b0^2}{3\,ep^6}-\frac{4\,b0}{3\,ep^5}\right)h0^2+\left(-\frac{2\,b0}{ep^2}+\frac{1}{ep}\right)h2\right\}$$

For decomposition of Dd3 we need {e0e, e1e, e2e} = {e0, e1, e2}/e0 - list . let us express this list in terms of e0 and h2.

e0 =.

h0 = (1 – e0) ep^2 / (2 b0 – ep);

{e0, e1, e2} / e0;

{e0e, e1e, e2e} = Normal[Series[%, {e0, 0, 12}]];

e2e

$$\frac{4\,e0\,(5\,b0^2 - b0\,ep)}{3\,ep^2\,(-2\,b0 + ep)^2} + \frac{-26\,b0^2 + b0\,ep}{3\,(2\,b0 - ep)^2\,ep^2} + \frac{2\,b0^2 + b0\,ep - 8\,b0^3\,h2 + 12\,b0^2\,ep\,h2 - 6\,b0\,ep^2\,h2 + ep^3\,h2}{e0\,(2\,b0 - ep)^2\,ep^2}$$

Discard from this expression , terms with factor h2 and the terms with pole 1/e0. Then

$$e2e = \frac{4\,e0\,(5\,b0^2 - b0\,ep)}{3\,ep^2\,(-2\,b0 + ep)^2} + \frac{-26\,b0^2 + b0\,ep}{3\,(2\,b0 - ep)^2\,ep^2};$$

$$e2ep = \frac{2\,b0^2 + b0\,ep}{(2\,b0 - ep)^2\,ep^2};$$

H^2 = Hh0x + Hh1 x^2 + Hh2 x^3 and e_y^2 = e0 x + e1 x^2 + e2 x^3 . Both expansions are related each other by the following equation

$$\frac{d}{d\epsilon} H\hat{}2 = (b0 - \epsilon)\,\frac{d}{d\epsilon}\,e_y^2$$

```
Clear[b1, e0, e1, e2, e3]

x1 = (b1 − x) (e0 + 2 e1 x + 3 e2 x^2);

Normal[Series[x1, {x, 0, 2}]];

{Hh0, Hh1, Hh2} = CoefficientList[%, x] {1, 1/2, 1/3}
```

$$\left\{b1\,e0,\ \frac{1}{2}\,(-e0 + 2\,b1\,e1),\ \frac{1}{3}\,(-2\,e1 + 3\,b1\,e2)\right\}$$

Take following list

```
{Hh0, Hh1, Hh2} / Hh0;

{Hh0H, Hh1H, Hh2H} = Collect[%, {e1, e2}];
```

Dd2 can be written in following form:

$\mathbf{Dd2 = h3\,\frac{ep}{h0\hat{}2} - \frac{ep}{h0}\,e2ep + Dd0 * Dd2d}$, **where** the first term is proportional to h3, the second term is due to 1/e0 -pole and the last is the other terms.

Seeks explicit expression for Dd2d which is ,in turn, of three parts: the first corresponds TM waves, the second part corresponds TE waves and third part describes imteractions between TM and TE waves..

Notation:

$$b11 = 2\,e1e - \frac{1}{2\,b1};\ b12 = -\frac{7}{6}\,e1e/b1 + e1e\hat{}2 + 2\,e2e;$$

$$h2 = 0;\ h1 = \frac{h0}{2\,ep} + \frac{2\,b0\,h0^2}{ep^3};$$

$$b1 = b0 - ep;\ b2 = b1 + b0;$$

$$b21 = h1/h0 + Aa1a;\ b22 = Aa1a\ h1/h0 + Aa2a;$$

e0 =.

Choose from Dd2d the terms which correspond to T E waves.

$$x1 = \frac{1}{8}(-b11^2 + 4\ b12);$$

Normal[Series[x1, {e0, 0, 10}]]

$$\frac{2\,e0\,(10\,b0^3 - 10\,b0^2\,ep + b0\,ep^2)}{3\,(b0 - ep)\,(2\,b0 - ep)^2\,ep^2} + \frac{-832\,b0^4 + 1568\,b0^3\,ep - 716\,b0^2\,ep^2 - 20\,b0\,ep^3 - 3\,ep^4}{96\,(b0 - ep)^2\,(2\,b0 - ep)^2\,ep^2}$$

{x10, x11} = CoefficientList[%, e0];

Choose from Dd2d the terms which correspond to T M waves.

$$x2 = \frac{1}{8}(3\,b21^2 - 4\,b22);$$

Normal[Series[%, {h0, 0, 10}]];

CoefficientList[%, h0]

$$\left\{\frac{-4\,b0^2 + 4\,b0\,ep + 9\,ep^2}{96\,ep^2\,(-b0 + ep)^2},\ \frac{4\,b0^2 - 5\,b0\,ep}{3\,(b0 - ep)\,ep^4},\ \frac{4\,b0^2}{ep^6}\right\}$$

{x20, x21, x22} = %;

h0 =.

$$e0 = 1 + \frac{ep - 2\,b0}{ep\hat{\ }2}\,h0;$$

The mixed terms in Dd2d are:

$$x3 = -\frac{1}{4}\,b11\,b21;$$

Normal[Series[%, {h0, 0, 10}]];

{x30, x31, x32} = CoefficientList[%, h0]

$$\left\{-\frac{1}{16\,(b0 - ep)^2},\ \frac{b0}{(b0 - ep)\,ep^3},\ -\frac{4\,b0^2}{ep^6}\right\}$$

Expressing TE terms in the term of h0

$$zz = \frac{1}{8}(-b11^2 + 4\ b12);$$

Normal[Series[%, {h0, 0, 10}]];

CoefficientList[%, h0]

$$\left\{\frac{-64\,b0^4 + 96\,b0^3\,ep - 4\,b0^2\,ep^2 - 28\,b0\,ep^3 - ep^4}{32\,(b0 - ep)^2\,(2\,b0 - ep)^2\,ep^2}, -\frac{2\,(10\,b0^3 - 10\,b0^2\,ep + b0\,ep^2)}{3\,(b0 - ep)\,(2\,b0 - ep)\,ep^4}\right\}$$

{z0, z1} = %;

x20 + x30 + z0;

Simplify[%];

g0 = %

$$\frac{b0\,(-52\,b0^2 + 28\,b0\,ep + 23\,ep^2)}{24\,(b0 - ep)\,ep^2\,(-2\,b0 + ep)^2}$$

x21 + x31 + z1;

Simplify[%];

g1 = %

$$-\frac{4\,b0^2}{(2\,b0 - ep)\,ep^4}$$

x22 + x32;

Simplify[%];

g2 = %

0

So, for Dd2d we have got explicit expression: Dd2d = g0 + g1 h0 + g2 h0^2

h0 =.;

Clear[h0, h1, h2];

$$b1 = b0 - ep;\ Dd0 = -ep\left(\frac{1}{h0} + \frac{ep - 2\,b0}{ep\,\hat{}\,2}\right);\ Dd1 = 0;$$

$$h1 = \frac{h0}{2\,ep} + \frac{2\,b0\,h0^2}{ep^3};$$

Let us imagine that the equation(see equation (15a) in the paper), in which Dd3 figures, has been written in the form, where terms with factor h3 were to the left side of equal sign and the other terms were to the right side of equals sign. If you omit the Dd3d terms and pole term then right side of the equation looks like

x1 = −(ep + x)^2 − (h0 x + h1 x^2) 2 b0 / (ep + x) −
(ep − 2 b0 + x + (ep + x) (Dd0 + Dd1 x)) (h0 + 2 h1 x) ;

Normal[Series[x1, {x, 0, 2}]]

$$\left(\frac{3\,b0\,h0}{ep^2}-\frac{12\,b0^2\,h0^2}{ep^4}\right)x^2$$

Note that, in this expansion there is only x^2 factor , which indicates the correctness of previous approximations.

Brx = % / x ^ 2;

$$e0 = 1+\frac{ep-2\,b0}{ep\,\hat{}\,2}\,h0;$$

Returning back to the right side of the equation, the discarded befor Dd2d terms and the pole term we wright

Br = Brx + ep ^ 2 e2ep + ep ^ 2 e0 (g0 + g1 h0 + g2 h0 ^ 2);

$$h2 = -\frac{h0}{2\,ep\,\hat{}\,2}\,Br;$$

Normal[Series[%, {h0, 0, 8}]]

$$-\frac{b0\,h0}{48\,ep^2\,(-b0+ep)}+\frac{(50\,b0^2-49\,b0\,ep)\,h0^2}{48\,ep^4\,(-b0+ep)}+\frac{4\,b0^2\,h0^3}{ep^6}$$

It is that we looking for.

$$h2 = -\frac{b0\,h0}{48\,ep^2\,(-b0+ep)}+\frac{(50\,b0^2-49\,b0\,ep)\,h0^2}{48\,ep^4\,(-b0+ep)}+\frac{4\,b0^2\,h0^3}{ep^6};$$

h0 =.

Dd2 is now calculated as

$$Dd3 = Dd0\,(g0+g1\,h0+g2\,h0\,\hat{}\,2)+h2\,\frac{ep}{h0\,\hat{}\,2}-\frac{ep}{h0}\,e2ep;$$

Normal[Series[%, {h0, 0, 8}]]

$$\frac{-2\,b0^2+b0\,ep}{16\,(b0-ep)\,ep^3}+\frac{b0}{16\,(b0-ep)\,ep\,h0}$$

$$Dd2 = \frac{-2\,b0^2+b0\,ep}{16\,(b0-ep)\,ep^3}+\frac{b0}{16\,(b0-ep)\,ep\,h0};$$

This result will be used in next steps of approximations.

Verification of the obtatned solution.

```
x1 = -(ep + x) ^ 2  - (h0 x + h1 x ^ 2) 2 b0 / (ep + x) -
       (ep - 2 b0  + x + (ep + x) (Dd0 + Dd1 x + Dd2 x ^ 2)) (h0 + 2  h1 x + 3 h2 x ^ 2) ;
```

Normal[Series[%, {x, 0, 2}]]

0