Influence of raindrop size distribution variations on the rain specific attenuation coefficients

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The raindrop size distribution is one of main term to evaluate microwave attenuation due to rainfall. In this paper, the lognormal distribution is introduced to represent the raindrop-size distribution based on the measurement performed at the Chungnam National University. Since the distribution parameters are strongly scattered around the fitted value, we developed a new statistical model for the lognormal distribution parameters. Based on the new statistical model for raindrop size distribution parameters we have estimated influence of raindrop size distribution variations on the rain specific attenuation coefficients.

I. INTRODUCTION

The attenuation due to rain is caused by two mechanisms: the absorption and the scattering by raindrops. The behaviours of these mechanisms depend on three factors of rainfall medium: the complex refractive index of water, the shape of raindrops and raindrop size distributions. In the present studies we have used the model for complex refractive index of water proposed by Liebe et al [1], and assumed the shape of raindrops are sphere. Natural raindrop size distributions are highly variable [3,4]. Therefore, to account the variability of raindrop size distributions we have developed statistical model for the parameters of lognormal distributions [2]. Then influence of the variations of raindrop size distributions on the rain specific attenuation coefficients are estimated using Monte Carlo simulations.

II. BACKGROUND

The rain specific attenuation of the microwave due to rain can be expressed [6]:

$$\gamma = 4.343 \cdot 10^3 \cdot \int_{D_{\min}}^{D_{\max}} Q_{ext} \frac{\pi D^2}{4} \cdot N(D) \cdot dD \qquad (1)$$

where $N(D)$ denotes raindrop size distribution. On the other hand, the rainfall rate is estimated as:

$$R = 6 \cdot \pi \cdot 10^{-4} \cdot \int_{D_{\min}}^{D_{\max}} D^3 \cdot v(D) \cdot N(D) \cdot dD \qquad (2)$$

where $v(D)$ represents raindrop terminal velocity [6].

The experimental measurements and theoretical calculations indicate very good correlations between the rainfall rate and the specific attenuation due to rain. Hence, for specific attenuation calculations used power regression equation

$$\gamma = \alpha \cdot R^{\beta} \qquad (3)$$

Therefore theoretical calculation of the specific attenuation due to rain leads to estimate the specific attenuation coefficients α and β. Usually these coefficients are calculated based on the equilibrium shape and average raindrop size distributions [6]. The lognormal distribution for raindrop-size distribution is formulated as [2];

$$N(D; N_0, \mu, \sigma) = \frac{N_0}{\sqrt{2 \cdot \pi} \cdot \mu \cdot \sigma} \cdot \left(\frac{D}{\mu}\right)^{-1} \cdot$$

$$\cdot \exp\left\{-\frac{1}{2} \cdot \left[\frac{1}{\sigma} \ln\left(\frac{D}{\mu}\right)\right]^2\right\} \qquad (4)$$

The parameters of the lognormal distributions at given rainfall are considered as random variables with definite distribution functions. Experimentally estimated histograms of the lognormal distribution parameters hints the parameters μ and σ could be described by normal, and N_0 - by lognormal distributions. Hence the distribution of the parameters $ln(N_0)$, μ and σ assessed as the 3-variates normal distribution with corresponding covariance matrix $\sum$, that is described as

$$p(x) = \frac{1}{(\sqrt{2\pi})^3 \cdot |\Sigma|^2} \exp\left[\frac{1}{2} \cdot (x - M)^T \cdot \Sigma^{-1} \cdot (x - M)\right]$$

$$M = \lfloor \mu_{\ln(N_0)},\ \mu_{\mu},\ \mu_{\sigma} \rfloor \qquad (5)$$

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$$\Sigma = \begin{pmatrix} \sigma_{\ln(N_0)} & Cov(\ln(N_0), \mu) & Cov(\ln(N_0), \sigma) \\ Cov(\ln(N_0), \mu) & \sigma_\mu & Cov(\mu, \sigma) \\ Cov(\ln(N_0), \sigma) & Cov(\mu, \sigma) & \sigma_\sigma \end{pmatrix}$$

III. THE STATISTICAL MODEL FOR RAINDROP SIZE DISTRIBUTIONS

All the analyses given in this paper are based on the data collected from July 2003 to November 2003 at the Chungnam National University. The Joss-Walvogel distrometer (RD-80, Distromet Co. Ltd) has been used for measurement. To reduce the sampling error we have chosen distrometer integration time to be 60 sec. The parameters N_0, μ and σ were estimated by the maximum likelihood estimation method. Then the parameters of the 3-variate normal distributions for $[N_0, \mu, \sigma]$ are estimated as function of rainfall rate by using robust estimation method. They are expressed

$$\mu_{\ln(N_0)} = 0.1326 \cdot \ln(R) + 6.541$$
$$\mu_\mu = \begin{cases} 0.09782 \cdot \ln(R) + 0.537; & if \; R \le 10 \\ 0.2511 \cdot \ln(R) + 0.1536; & if \; R > 10 \end{cases}$$
$$\mu_\sigma = \begin{cases} 0.0829 \cdot \ln(R) + 0.27; & if \; R \le 10 \\ -0.003323 \cdot \ln(R) + 0.4641; & if \; R > 10 \end{cases}$$
$$\sigma_{\ln(N_0)} = -0.1183 \cdot \ln(R) + 0.801$$
$$\sigma_\mu = 0.02159 \cdot \ln(R) + 0.08213 \tag{6}$$
$$\sigma_\sigma = -0.002328 \cdot \ln(R) + 0.0623$$
$$Cov(\ln(N_0), \mu) = 0.00692 \cdot \ln(R) - 0.07$$
$$Cov(\ln(N_0), \sigma) = 0.007371 \cdot \ln(R) - 0.02344$$
$$Cov(\mu, \sigma) = -0.002141 \cdot \ln(R) + 0.001834$$

The algorithm for generating the desired 3-varaites $[N_0, \mu, \sigma]$ random vector is as follows:

1. Generate $[z_1, z_2, z_3]$ vector as Normal (0,l) random variates.
2. Calculate $[\ln(N_0), \mu, \sigma]^T = [\mu_{\ln(No)}, \mu_\mu, \mu_\sigma]^T + \sum^* [z_1, z_2, z_3]$;
3. Return $[\exp(\ln(N_0)), \mu, \sigma]^T$.
4. Generate raindrop sizes using parameters $[N_0, \mu, \sigma]$

To test the proposed statistical model for raindrop size distribution, the Monte Carlo simulations were carried out. For example one realization of Monte Carlo simulation results are illustrated on Figure 1. Comparing of this scatter-gram to those found in experiment gives excellent agreement.

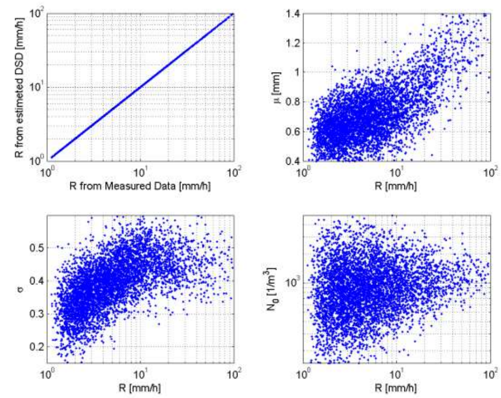

Figure 1. Scattergram of the lognormal distribution parameters.

IV. THE INFLUENCE OF THE RAINDROP SIZE DISTRIBUTIONS VARIATIONS ON THE RAIN SPECIFIC ATTENUATION COEFFICIENTS

Using our statistic model for raindrop size distribution, we have carried out Monte Carlo simulation to reveal the influence of the raindrop size distribution variations on the rain specific attenuation coefficients. For each simulated raindrop size distributions we have calculated the rain specific attenuation coefficients. These coefficients variations against frequency are shown Figure 3. Here, for each frequencies scattering of each coefficients are plotted using boxplot function of MATLAB. It has been found the specific attenuation coefficients more strictly influenced by raindrop size distributions than raindrop shape.

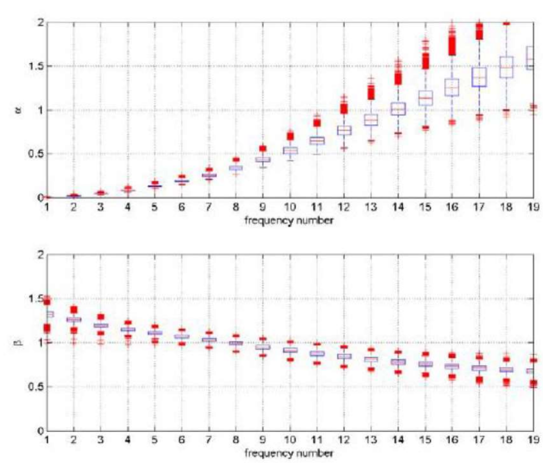

Figure 2. The influence of the raindrop size distribution variations on the rain specific attenuation coefficients.

The effect of raindrop size distribution variations on the β coefficient almost not depend on the frequency, whereas, the effect of raindrop size distribution variations on the α coefficient increases with frequency. Hence, as frequency increases one should care about the raindrop size distribution variations.

V. CONCLUSIONS

In this paper, we introduced new models for raindrop-size distributions, based on the measurement performed at the Chungnam National University. It is concluded that the lognormal distribution is the most adequate candidate for describing raindrop-size distributions than other distribution models. Also, a new statistical model for the lognormal distribution parameters was developed to take advantage of the Monte Carlo method. In the framework of Mie scattering theory the influence on raindrop size distribution variations on the rain specific attenuation coefficients are estimated by Monte Carlo simulations. Simulation result shows the specific attenuation coefficients more strictly influenced by raindrop size distributions than raindrop shape.

VI. REFERENCES

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