

# Two-body problems in the orthogonal condition model

M.Odsuren<sup>1,\*</sup>, G.Khuukhenkhuu<sup>1</sup>, S.Davaa<sup>1</sup>, K.Katō<sup>2</sup>, A. Sarsembayeva<sup>3</sup>

<sup>1</sup>*School of Engineering and Applied Sciences and Nuclear Research Center, National University of Mongolia, Ulaanbaatar 210646, Mongolia*

<sup>2</sup>*Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan*

<sup>3</sup>*School of Industrial Engineering after A. Burkitbaev, Satbayev University, Almaty, 0500013, Kazakhstan*

In this work we investigate the 0+, 2+, 4+, 6+, 8+ and 10+-resonant states of the  $\alpha$ - $\alpha$  system and each advantage of Gaussian and Harmonic Oscillator basis wave functions in the complex scaled orthogonal condition model (CSOCM).

## INTRODUCTION

The complex scaling method (CSM) [1-4] and the orthogonal condition model (OCM) [5] have been successfully utilized in the description of resonance states in light nuclei. In this work, we apply the CSOCM [4,6] to the  $^8\text{Be}$  and investigate two-body resonances for  $\alpha$ - $\alpha$  system. We calculate resonance energies in the complex energy plane applying the CSM to the relative motion between two  $\alpha$ -clusters. From the viewpoint of a microscopic description of the relative motion between the  $\alpha$ - $\alpha$  clusters it is important to take into account the Pauli exclusion principle in the inter-cluster motion of nucleons. The Gaussian and Harmonic Oscillator wave functions are applied. The calculation procedure of using the Pauli principle are different in these basis functions, however, the same results for two-body system are expected. Our calculated results of the resonance energy and decay width are satisfactorily in agreement with experimental data for the  $J^\pi=0^+$ ,  $2^+$  and  $4^+$  states [7]. The purpose of this work is to calculate experimentally unknown  $6^+$ ,  $8^+$  and  $10^+$  higher excited states of  $^8\text{Be}$  system.

## COMPLEX SCALING METHOD

In the last quarter century, a remarkable development in the description of resonances in quantum many-body systems has been realized through application of the CSM.

Originally, the CSM was proposed by Aguilar, Combes, and Balslev in 1971 [1]. Simon advocated this method as a direct approach of obtaining many-body resonances. The use of “direct” implies that the resonance wave functions are directly obtained with complex energy eigenvalues of the quantum many-

body system by solving an eigenvalue problem of the complex-scaled Schrodinger equation,  $H^\theta \Psi^\theta = E^\theta \Psi^\theta$  with a real scaling angle  $\theta$ . In the CSM, we take the imaginary value  $i\theta$  as a parameter of the transformation.

The CSM has been proposed to solve the resonance states in the similar way as bound state problems. In the CSM, the distance of the relative coordinate is rotated as  $r \rightarrow r e^{i\theta}$  in the complex coordinate plane by introducing a real parameter  $\theta$ . Therefore, the Schrödinger equation

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \quad (1)$$

is rewritten as

$$\hat{H}(\theta)|\Psi^\theta\rangle = E^\theta|\Psi^\theta\rangle, \quad (2)$$

where  $\hat{H}(\theta)$  and  $\Psi^\theta$  are the complex scaled Hamiltonian and the wave function, respectively.  $U(\theta)$  operates on a function  $\Psi$ , that is,

$$\Psi^\theta = U(\theta)\Psi(r) = e^{\frac{3}{2}i\theta}\Psi(r e^{i\theta}) \quad (3)$$

The eigenvalues and eigenstates are obtained by solving the complex scaled Schrodinger equation Eq.(2). The eigenvalues of resonance states are found as  $E^\theta = E_r - i\Gamma_r/2$ , where  $E_r$  is resonance energy and  $\Gamma_r$ -width of the resonant state. More detailed explanation of the CSM is given in Refs.[1, 2]. The complex scaled Hamiltonian of inter cluster motion is given by

$$\hat{H}(\theta) = U(\theta)\hat{H}U^{-1}(\theta) \quad (4)$$

## TWO BODY INTERACTION

For the alpha-alpha system the Hamiltonian is expressed as

---

\* Electronic address: odsurenn@gmail.com

$$\hat{H} = \sum_{i=1}^2 \hat{T}_i - \hat{T}_{c.m.} + V_{\alpha\alpha}^{Nucl}(r) + V_{\alpha\alpha}^{Coul}(r).$$

As mentioned at the beginning, in this work we use two different basis sets as follows:  
(i) A Gaussian basis for the radial part is given as

$$\phi_i^l(r) = N_i^l r^l \exp\left(-\frac{1}{2b_i^2} r^2\right) Y_{lm}(r).$$

Here  $i = 0, 1, 2, \dots$ , and  $N_i^l$  is normalization constants expressed as  $N_i^l = \frac{1}{b_i^{l+3/2}} \left\{ \frac{2^{l+2}}{(2l+1)!! \sqrt{\pi}} \right\}^{1/2}$  and  $b_i$  is

the size parameter of Gaussian function described as  $b_i = b_0 \gamma^{i-1}$ . Where  $b_0$  and  $\gamma$  are the first term and a common ratio in the geometric progression, respectively.

(ii) Harmonic oscillator wave function for radial part is

$$\phi_{nl}(r) = N_l^n \left(\frac{r}{b_F}\right)^l L_n^{l+1/2} \left(\left(\frac{r}{b_F}\right)^2\right) \exp\left(-\frac{1}{2b_F^2} r^2\right) Y_{lm}(r)$$

here  $L_n^{l+1/2}$  are Laguerre polynomials for the angular momentum  $l$  and  $N_l^n$  denotes the normalization constants as given by  $N_l^n =$

$$\left\{ \frac{2\Gamma(n+1)}{b_F^3 \Gamma(l+n+\frac{3}{2})} \right\}^{1/2}.$$

The size parameter of relative motion of two alpha-cluster  $b_F$  is taken as 0.967 fm which corresponds to a single particle size parameter  $b_0 = 1.3975$  fm employed to fit the observed r.m.s. radius of  ${}^4\text{He}$ . In the case (i), we introduce the Pauli-potential  $V_{\alpha\alpha}^P(r) = \lambda |\chi_F\rangle \langle \chi_F|$ , where the strength  $\lambda$  is chosen as  $10^7$  MeV, which is enough to push up the Pauli-forbidden states into the unphysical energy region.

## RESULTS AND DISCUSSIONS

In the numerical calculation, we have used two different basis set: (i) Gaussian basis function, and (ii) harmonic oscillator wave function. In Eq.5, the Buck [8] and folding [9] potentials are applied for the Gaussian basis function, but also the folding potential is employed in the harmonic oscillator wave function. According to the Buck-potential, the Pauli-forbidden states need not involve on the alpha-alpha system because of the Pauli principle effect is estimated by an appropriate choice of alpha-alpha potential. However, it is important to take into account of the forbidden states when we use the folding potential of the effective nuclear interaction.

From Eq.(2) the eigenvalues are obtained distributions of which on the complex energy plane are shown Figs. 1-2. Figs. 1-2 show the complex energy eigenvalues of  $2^+$  and  $4^+$  states which are obtained by Buck and folding potentials for Gaussian basis at different  $\theta$  on the complex energy plane. The resonance energy solution must be stationary for changing the values of  $\theta$  as explained in Ref. [6].

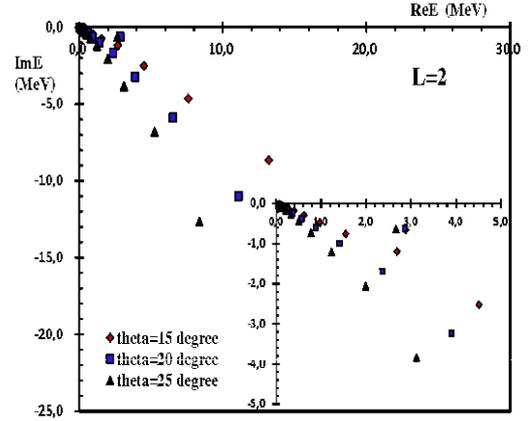


Figure 1. The resonance eigenvalues at  $J^\pi=2^+$  for the different  $\theta$ . Here Buck-potential is used for Gaussian basis.

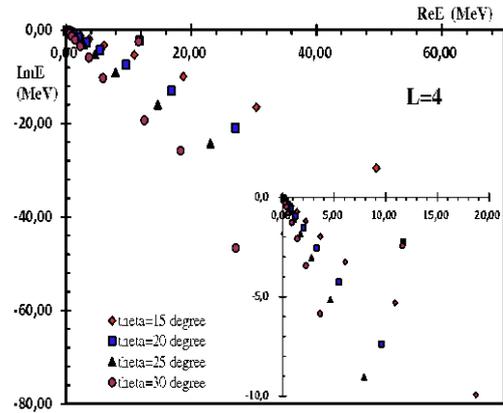


Figure 2. The resonance eigenvalues at  $J^\pi=4^+$  for the different  $\theta$ . The folding potential is used for Gaussian basis.

According to this explanation, we can see that for different  $\theta$  segregated energy points are observed, but also these are almost unchanged the position by various  $\theta$  on the complex energy plane (see Figs. 1 and 2). Fig. 1-2 show that there is significant energy point segregation around the location resonance state at the complex scaled plane.

In addition, Figure 1 and 2 present the outcome of the Gaussian basis function. Here Buck and folding potential parameters are applied at  $\theta = 15^\circ, 20^\circ, 25^\circ$  and at  $\theta = 15^\circ, 20^\circ, 25^\circ, 30^\circ$  on the complex energy plane, respectively. Furthermore, in

order to describe accurately the energies and widths for resonance state should be to consider the  $\theta$ - and  $b$ -trajectories. In Figs. 3-4 are displayed eigenvalues which were calculated by  $\theta$ - and  $b$ -trajectories.

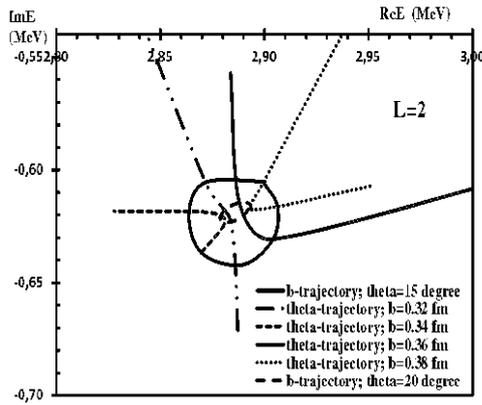


Figure 3. The  $\theta$  and  $b$ -trajectory at  $J^\pi=2^+$ . The Buck-potential and Gaussian basis are used.

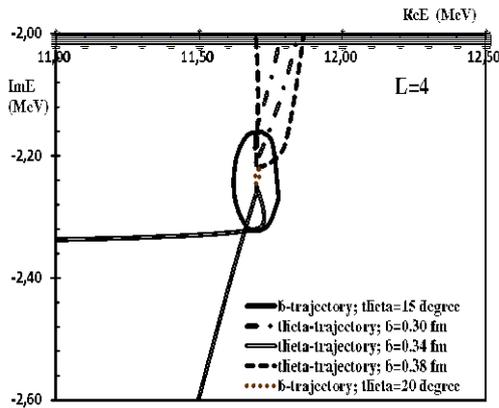


Figure 4. The  $\theta$  and  $b$ -trajectory at  $J^\pi=4^+$ . The folding potential and Gaussian basis are used.

Therefore, the  $b$ -trajectories shapes are displayed as circles and  $\theta$ -trajectories shapes like curves.  $b$ -trajectories are performed two times for each different potential parameters as shown Fig. 3-4 and  $\theta$ -trajectories are calculated 4 and 3 times for Buck and folding potentials, respectively. Moreover, we chose the steps of  $b$ -trajectories by  $b = b_0 + 0.1\kappa$  here  $\kappa = 1, 2, \dots, 10$  which were calculated by two methods: changing parameter  $b$  and  $\theta$  is fixed for each  $b$ -trajectories, and by the same  $b$  for every  $\theta$ -trajectories, here  $\theta$  is changing parameter and taken by  $\theta = \theta_0 + \kappa$  where  $\kappa = 0, 2, 4, \dots, 20$ .

It can be seen from Figs. 3-4, the resonance states are accurately described when the behavior of the  $\theta$ - and  $b$ -trajectories are created well. The calculated result of harmonic oscillator wave function for folding potential is displayed in Fig. 5.

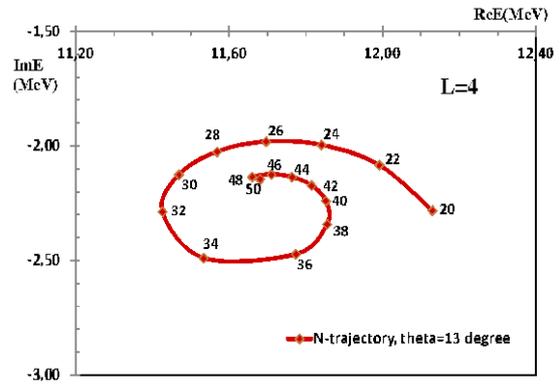


Figure 5. The  $N$ -trajectory at  $J^\pi=4^+$ . The folding potential and harmonic oscillator wave function are used.

Fig. 5 shows the  $N$ -trajectories at  $\theta = 13$  and  $N = N_0 + k$  here  $k = 0, 2, 4, \dots, 30$ . The accurate values of resonances are taken into account by  $\theta$ - and  $N$ -trajectories for harmonic oscillator wave function. The spiral curve represents the  $N$ -trajectory followed by the basis states when its size increases. The approach of the energy point is round  $N = 46 - 50$ .

The results of the calculated energies with decay widths for  $0^+, 2^+, 4^+, 6^+, 8^+$  and  $10^+$  states of  ${}^8\text{Be}$ , experimental data and two different potential parameters are included in Table I. The experimental data are taken from Ref. [7]. Calculated results are obtained by using the various bases within the Gaussian and harmonic oscillator wave functions. The low-lying calculated states are comparable with observed data [7]. However, there is a slight difference of energy at the  $0^+$  state between the calculated result for folding potential [9] and measured data. Furthermore, there is a difference between the calculated decay width of Gaussian basis and harmonic oscillator wave functions at the  $0^+$  state. In order to clarify a reason of this difference, it may necessary to check the convergence of solutions and increase the number of employed basis functions. In addition, the obtained results by Gaussian basis function and Buck potential are 33.4 MeV (37.2 MeV) and 51.5 MeV (92.4 MeV) energies (widths) at the  $6^+$  and  $8^+$  states, but as we know no experimental evidence to support this calculated results, however, these results are indicated a good agreement with other recent computed results [10] which are 34.38 MeV (37.19 MeV) and 53.65 MeV (93.74 MeV) at the  $6^+$  and  $8^+$  states, respectively. Furthermore, the higher excited state  $10^+$  is computed by two bases functions

applying both potential parameters. The calculated results are in good agreement with each other.

TABLE 1. The experimental and calculated resonance energies and widths of the  $^8\text{Be}$ .

	EXP		Gaussian Basis function				Harmonic oscillator wave function	
	[7]		$E_r(\text{MeV})$	$\Gamma_r(\text{MeV})$	$E_r(\text{MeV})$	$\Gamma_r(\text{MeV})$	$E_r(\text{MeV})$	$\Gamma_r(\text{MeV})$
0+	$9.1 \times 10^{-2}$	$6.8 \times 10^{-6}$	$9.13 \times 10^{-2}$	$\sim 10^{-6}$	$6.41 \times 10^{-1}$	$3.8 \times 10^{-5}$	$6.06 \times 10^{-1}$	$3.0 \times 10^{-3}$
2+	3.132	1.5	2.75	1.24	3.01	1.2	2.90	1.4
4+	11.49	3.5	11.78	3.56	11.75	4.4	11.7	4.4
6+	-	-	33.4	37.2	30.5	35.7	30.5	36.8
8+	-	-	51.5	92.4	51.6	120	51.6	120
10+	-	-	70.7	160	70.0	180	70.0	180
$V_0$			122.6225		106.09			
$\beta$ (fm $^{-2}$ )			0.22 [8]		0.2009 [9]			
$\alpha$ (fm $^{-1}$ )			0.75		0.5972			

## SUMMARY

In this work we have presented different methods to calculate resonance state in the two-body system by CSOCM. Moreover, it can be seen that the different potential parameters able on both Gaussian and harmonic oscillator wave functions. The  $\theta$ ,  $b$ - and  $N$ -trajectories are performed in order to determine the resonance states for different method.

The methods well explain all obtained resonance states of  $^8\text{Be}$  except the width for the  $0^+$  state calculated by harmonic oscillator wave function. Furthermore, the results of the harmonic oscillator basis functions show that we need to employ a very large size basis functions in order to obtain converged resonance energies and widths.

It is remarkable that  $6^+$ ,  $8^+$  and  $10^+$  higher excited energies are calculated and broad decay widths are predicted. In connection with the broad decay widths these higher excited states may not able to observe as well defined resonances in experiment.

## REFERENCES

- [1] Y.K. Ho, Phys. Rep.99,1(1983).
- [2] T. Myo, Y. Kikuchi, H. Masui, K. Katō, Prog. Part. Nucl.Phys. 79, 1 (2014).
- [3] M. Odsuren, K. Kato, M. Aikawa, T. Myo, Phys. Rev. C89, 034322 (2014).
- [4] M. Odsuren, K. Kato, M. Aikawa, Nucl. Data Sheets 120, 126 (2014).
- [5] S. Saito, Prog. Theor. Phys.40, 1968, pp.893-894; 41, 1969, pp.705-722; Prog. Theor. Phys. Suppl. 62, 11 (1977).
- [6] A. T. Kruppa, K. Katō, Prog. Theor. Phys. 84, 1145 (1990).

- [7] F. Ajzenberg-Selove, Nucl. Phys. A 490, 1 (1988).
- [8] B. Buck, H. Friedrich, C. Wheatley, Nucl. Phys, A 275, 246 (1977).
- [9] E. W. Schmid, K. Wildermuth, Nucl. Phys. 26, 463 (1961).
- [10] E. Garrido, A. S. Jensen, D. V. Fedorov, Phys. Rev. C. 88 024001 (2013).