Resonance states in the simple schematic two-body model

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The aim of this work is to obtain resonance states in the simple schematic two-body model. We take up two-body systems, which are described by Schrodinger equation. To obtain resonance states, we apply a simple schematic potential and complex scaling method (CSM). In the CSM, the resonance wave functions are obtained as eigenstates together with bound states by carrying out the diagonalization of the complex scaled Hamiltonian. By finding eigenvalues, we can show the distribution of resonance states in the complex energy plane. As a result, we obtain 5 and 4 resonance states for $J^\pi = 0^+, 1^-$ states, respectively.

INTRODUCTION

During the last several decade resonance problems have covered an important and crucial research area in nuclear physics. Recently, it has attracted much attention that the complex scaling method (CSM) [1-2] is successfully utilized for description of many-body resonant states in light and middle mass nuclei. Although many problems have been solved so far, but further researches are required still.

In this study, the complex scaling method is applied to a simple schematic two-body model [3] and its reliability is confirmed. For this purpose, several resonance states of $J^\pi = 0^+$ and 1^- partial waves are investigated using the simple schematic potential.

COMPLEX SCALING METHOD

In the last quarter century, a remarkable development in the description of resonances in quantum many-body systems has been realized through application of the CSM.

Originally, the CSM was proposed by Aguilar, Combes, and Balslev in 1971 [1]. Simon advocated this method as a direct approach of obtaining many-body resonances. The use of "direct" implies that the resonance wave functions are directly obtained with complex energy eigenvalues of the quantum many-body system by solving an eigenvalue problem of the complex-scaled Schrodinger equation, $H^\theta \Psi^\theta = E^\theta \Psi^\theta$ with a real scaling angle θ. In the CSM, we take the imaginary value $i\theta$ as a parameter of the transformation.

The CSM has been proposed to solve the resonance states in the similar way as bound state problems. In the CSM, the distance of the relative coordinate is rotated as $r \longrightarrow re^{i\theta}$ in the complex coordinate plane by introducing a real parameter θ. Therefore, the Schrödinger equation

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \quad (1)$$

is rewritten as

$$\hat{H}(\theta)|\Psi^\theta\rangle = E^\theta|\Psi^\theta\rangle, \quad (2)$$

where $\hat{H}(\theta)$ and Ψ^θ are the complex scaled Hamiltonian and the wave function, respectively. $U(\theta)$ operates on a function Ψ, that is,

$$\Psi^\theta = U(\theta)\Psi(r) = e^{\frac{3}{2}i\theta}\Psi(re^{i\theta}). \quad (3)$$

The eigenvalues and eigenstates are obtained by solving the complex scaled Schrodinger equation Eq.(2). The eigenvalues of resonance states are found as $E^\theta = E_r - i\Gamma_r/2$, where E_r is resonance energy and Γ_r-width of the resonant state. More detailed explanation of the CSM is given in Refs.[1, 2]. The complex scaled Hamiltonian of inter cluster motion is given by

$$\hat{H}(\theta) = U(\theta)\hat{H}U^{-1}(\theta). \quad (4)$$

RESULTS AND DISCUSSIONSSIONS

The Hamiltonian of the present model is given as

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r), \quad (5)$$

where

$$V(r) = -8.0\exp(-0.16r^2) + 4.0\exp(-0.04r^2)\ . \quad (6)$$

For simplicity, we put $\frac{\hbar^2}{\mu} = 1$ (MeV fm2). This potential introduced in Ref. [3] has an attractive

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pocket in a short range but a repulsive barrier at a large distance. Putting Eq.(6) in Eq.(5), we solve the Schrödinger equation (Eq.(2)). To solve the Eq. (2), we employ the Gaussian basis functions given as

$$u_i(\hat{r}) = N_l(b_i) r^l \exp\left(-\frac{1}{2b_i^2} r^2\right) Y_{lm}(\hat{r}), \qquad (7)$$

where the range parameters are given by a geometric progression as $b_i = b_0 \gamma^{i-1}$, $i = 1,2,\dots,N$.

In this calculation, we apply $N = 20$ and employ the optimal values of b_0 and γ to obtain stationary resonance solutions.

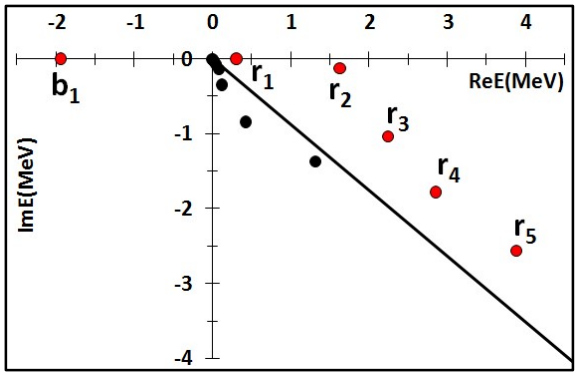

Figure 3. Distribution of energy eigenvalues of the $J^\pi = 0^+$ wave. Symbols (b1) and (r1, r2, r3, r4, r5) represent bound and resonance solutions, respectively. We here employ scaling angle $\theta = 15^0$. The solid line from the origin indicates the so-called 2θ line describing the branch cut.

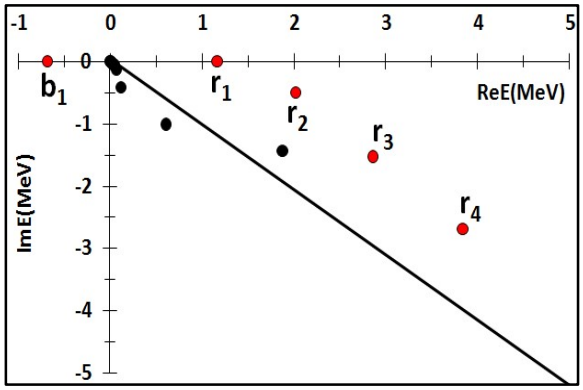

Figure 4. Distribution of energy eigenvalues of the $J^\pi = 1^-$ wave. Symbols (b1) and (r1, r2, r3, r4) represent bound and resonance solutions, respectively. We here employ scaling angle $\theta = 15^0$. The solid line from the origin indicates the so-called 2θ line describing the branch cut.

To obtain stationary values for the parameters of resonances, we apply the so-called b and θ trajectory methods. Using a property that is b and θ trajectories should be orthogonal to each other, we can easily determine the stationary point of the resonance energy with high precision by drawing these trajectories for the obtained eigen-energies [4].

We fixed Gaussian basis function's length parameter b from 0.15 to 0.25 and θ from 5 to 25 for the θ trajectory. The b trajectory was found to be roughly a circle. The true solution for resonance energy should be inside this circle, because the b trajectory is orthogonal to the θ trajectory. Therefore, we determined resonance energy as the center of the circle.

When we plot both the b and θ trajectories, we obtain an accurate estimate of the resonance position.

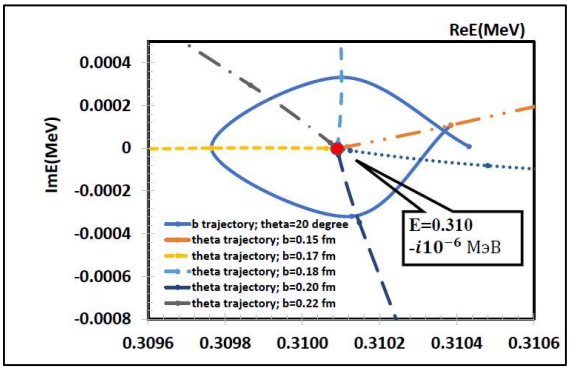

Figure 1. Resonance energy eigenvalues of the $J^\pi = 0^+$ wave by drawing b and θ trajectories.

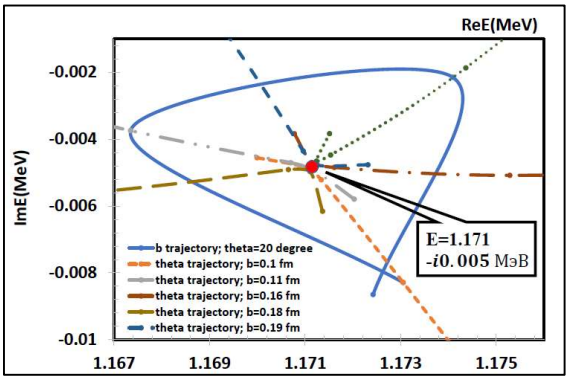

Figure 2. Resonance energy eigenvalues of the $J^\pi = 1^-$ wave by drawing b and θ trajectories.

In Tables 1 and 2, we show numerical values of the calculated bound and resonant states for the $J^\pi = 0^+$ and 1^- waves respectively, and compare to the results (left) that takes from the Ref. [5].

Table 1. Bound and resonance states energies with decay widths calculated for the $J^\pi = 0^+$ wave.

0^+wave*		0^+ wave	
E(MeV)	State	E(MeV)	State
-1.928	Bound	-1.928	Bound
0.310-$i10^{-6}$	Resonance	0.310-$i10^{-6}$	Resonance

1.632-i0.123	Resonance	1.633-i0.123	Resonance
2.249-i1.040	Resonance	2.249-i1.075	Resonance
2.854-i2.570	Resonance	2.850-i1.800	Resonance
		3.875-i2.575	Resonance

* From previous data [5]

Table 2. Bound and resonance energies with decay widths calculated for the $J^{\pi} = 1^{-}$ state.

$\mathbf{1}^{-}$ wave*		$\mathbf{1}^{-}$ wave	
E(MeV)	State	E(MeV)	State
-0.675	Bound	-0.675	Bound
1.171-i0.005	Resonance	1.171-i0.005	Resonance
2.031-i0.489	Resonance	2.018-i0.493	Resonance
2.832-i1.199	Resonance	2.830-i1.510	Resonance
3.934-i1.788	Resonance	3.655-i2.500	Resonance

* From previous data [5]

It can be seen that from Tables 1 and 2, two calculated results are similar to each other.

SUMMARY

In this study, we employed the simple potential model which gives a bound and several resonance states for $J^{\pi} = 0^{+}$and 1^{-} waves. Present calculated results are compared with the previous calculated result and we obtained both results are similar to each other.

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