Systematics of alpha-active nuclides and on the validity of the Geiger-Nuttall law

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Geiger-Nuttall law, which describes a dependence of the disintegration constant on the range of α-particles, was deduced using the Gamow theory describing the passage of the α-particles through the Coulomb barrier by the quantum mechanical tunneling effect. Ground-to-ground state α-transitions for natural and artificial α-active nuclides were analyzed utilizing the Geiger-Nuttall rule. From rough analysis five group-like branches on the dependence of α-decay half-lives on α-particle energy was observed. Detailed analysis shows that precise linear dependence of the logarithm of α-decay half-lives on the reciprocal of square root of the α-particle energy for even-even isotopes of the U, Pu and Cm there are. However, for some even-even isotopes of the Po, Ra and Th regular behaviour of mass numbers was broken. This non-regularity of the mass numbers on the Geiger-Nuttall line is explained by the nuclear shell model.

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I. INTRODUCTION

The alpha decay is a disintegration of the radioactive nucleus which emits an α-particle consisting of two protons and two neutrons. In 1911 Geiger and Nuttall established [1] an empirical law which describes a dependence of the disintegration constant on the range of α-particles. Energy of the outgoing α-particle is usually lower than potential energy of the daughter nucleus [2]. Although from the view point of the classical mechanics it is unclear how alpha particle can overcome from the nuclear potential, Gamow theory [3,4] can describe the passage of α-particles through the Coulomb potential barrier by the quantum-mechanical tunneling effect.

In this work the Geiger-Nuttall law deduced from the Gamow theory and ground-to-ground state α-decay data were systematically analyzed using this law and the nuclear shell model deductions.

II. THEORETICAL BACKGROUND

The Geiger and Nuttall law [1] relates the decay constant of a radioactive isotope with the range of the α-particle as following:

$$\log(\lambda) = A \log R_\alpha + B . \qquad (1)$$

Here: λ is the disintegration constant, $\lambda = 0.693/T_{1/2}$, where $T_{1/2}$ is the half-life; R_α is the range of the α-particle, $R_\alpha \sim E_\alpha^n$ where E_α is the α-particle energy; A and B are the constants. Then, the Geiger-Nuttall law can be rewritten as following:

$$\ln(T_{1/2}) = a \frac{1}{\sqrt{E_\alpha}} + b . \qquad (2)$$

The formula (2) can be deduced from Gamow theory [3-5]. Penetration probability of α-particle through potential barrier is determined by [5,6]:

$$\ln(T_\alpha) = -2 \int_R^{R_0} \sqrt{\frac{2m_\alpha (V(r) - E_\alpha)}{\hbar^2}} dr . \qquad (3)$$

Here: m_α is the mass of the α-particle; $V(r)$ is the potential energy of the daughter nucleus; R and R_o are the inner and outer classical turning points, respectively (Fig.1).

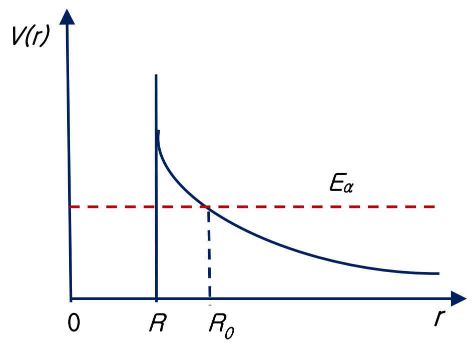

Fig.1. Alpha-particle penetration through the potential barrier.

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The inner classical turning point, R, can be obtained for square potential well as a daughter nuclear radius:

$$R = r_0 A_D^{1/3} \quad (4)$$

where: A_D is the mass number of the daughter nucleus and $r_0=1.25\cdot10^{-13}cm$. For the potential energy of the daughter nucleus, $V(r)$, we can use the Coulomb potential as a first approximation:

$$V(r) = \frac{2Ze^2}{r} \quad (5)$$

where: e is the elementary charge; and Z is the proton number of the daughter nucleus. Then, from Fig.1 the outer classical turning point can be determined from following expression:

$$\frac{2Ze^2}{R_0} = E_\alpha . \quad (6)$$

So, the formula (3) can be rewritten in the form

$$\ln(T_\alpha) = -2\int_R^{R_0} \sqrt{\frac{2m_\alpha\left[\frac{2Ze^2}{r} - E_\alpha\right]}{\hbar^2}}dr . \quad (7)$$

The following simple substitutions are used to calculate the integral in Eq.(7):

$$x = \frac{r}{R_0} \text{ and } x_0 = \frac{R}{R_0}. \quad (8)$$

Then, from the expression (7) can be gotten following formula:

$$\ln(T_\alpha) = -\frac{4e^2 Z}{\hbar}\sqrt{\frac{2m_\alpha}{E}}\int_{x_0}^{1}\sqrt{\frac{1}{x}-1}dx \quad (9)$$

The integral in Eq.(9) can be taken as follows:

$$\int_{x_0}^{1}\sqrt{\frac{1}{x}-1}dx = \int_{0}^{1}\sqrt{\frac{1}{x}-1}dx - \int_{0}^{x_0}\sqrt{\frac{1}{x}-1}dx \approx$$

$$\approx \int_0^1\sqrt{\frac{1}{x}-1}dx - \int_0^{x_0}\sqrt{\frac{1}{x}}dx = \int_0^1\sqrt{\frac{1}{x}-1}dx - 2\sqrt{x_0}. \quad (10)$$

Here the approximation of $x_0 << 1$ was used. If we use the substitution $x=sin^2\theta$ the integral in Eq.(10) is taken as following:

$$\int_0^1\sqrt{\frac{1}{x}-1}dx = 2\int_0^{\pi/2}\cos^2\theta d\theta = \frac{\pi}{2}. \quad (11)$$

So, the integral in (9) is given by

$$\int_{x_0}^{1}\sqrt{\frac{1}{x}-1}dx \approx \frac{\pi}{2} - 2\sqrt{\frac{R}{R_0}} . \quad (12)$$

Then, from Eqs.(7), (9) and (12) the following formula can be obtained

$$\ln(T_\alpha) \approx -\frac{2\pi e^2 Z}{\hbar}\sqrt{\frac{2m_\alpha}{E_\alpha}} + \frac{8}{\hbar}\sqrt{e^2 ZRm_\alpha} . \quad (13)$$

Taking into account an α-clustering effect, the disintegration constant for α-decay can be expressed as following:

$$\lambda = \phi_\alpha f_\alpha T_\alpha , \quad (14)$$

where: ϕ_α is the α-clustering factor; f_α is the collision frequency of the α-particle in the potential barrier of the daughter nucleus. In the case of one body approximation [7] the α-clustering factor can be assumed as f_α=1. Then, the Eq.(14) can be rewritten in the form

$$\lambda = \frac{0.693}{T_{1/2}} = f_\alpha T_\alpha . \quad (15)$$

The collision frequency of the α-particle can be obtained as

$$f_\alpha = \frac{\upsilon_\alpha}{2R} = \frac{\sqrt{2E_\alpha / m_\alpha}}{2R} . \quad (16)$$

From Eqs.(15) and (16) the half-life is given by

$$T_{1/2} = \frac{2R\cdot 0.693}{\sqrt{2E_\alpha / m_\alpha}}\cdot\frac{1}{T_\alpha} . \quad (17)$$

So, from Eqs.(13) and (17) can be got following expression

$$\ln(T_{1/2}) = \ln\frac{2R\cdot 0.693}{\sqrt{2E_\alpha / m_\alpha}} - \frac{8}{\hbar}\sqrt{e^2 ZRm_\alpha} + \frac{2\pi e^2 Z}{\hbar}\sqrt{\frac{2m_\alpha}{E_\alpha}} . \quad (18)$$

Then, the Eq.(18) can be rewritten in the following form

$$\ln(T_{1/2}) = a\frac{1}{\sqrt{E_\alpha}} + b , \quad (19)$$

where:

$$a = \frac{2\pi e^2 Z}{\hbar}\sqrt{2m_\alpha} \quad (20)$$

and

$$b = \ln\frac{2R\cdot 0.693}{\sqrt{2E_\alpha / m_\alpha}} - \frac{8}{\hbar}\sqrt{e^2 ZRm_\alpha} . \quad (21)$$

It can be seen that the formula (19) is the same as the expression (2) which was directly written from the Geiger and Nuttall law (1). It should be noted that α-particle energy under logarithm is included in the parameter b which can be considered almost constant in comparison with $1/\sqrt{E_\alpha}$ in the Eq.(19). Also, the proton number Z in Eqs.(20) and (21) can be taken as an effective and average value for all considered nuclides. Thus, the Eq.(19) will be utilized for systematical analysis of known experimental data of the α-decay.

III. RESULTS OF ANALYSIS AND DISCUSSION

Decay data of the ground-to-ground state α-transitions for over 450 natural and artificial alpha-active nuclides [8-10] including rare-earth and super-heavy elements were analyzed using the Geiger-Nuttall law (19). The dependence of the logarithm of α-decay half-lives, $T_{1/2}$ (sec), on the reciprocal of square root of the α-particle energy, E_α(MeV), for studied isotopes is shown in Fig.2.

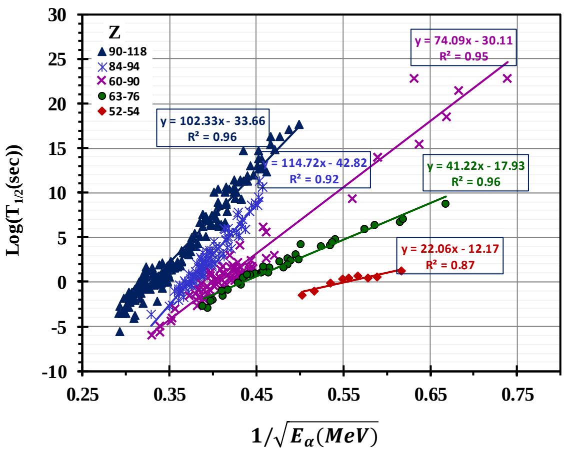

Fig. 2. The logarithm of α-decay half-lives versus the reciprocal of square root of the α-particle energy.

From the preliminary and rough analysis, it was seen that five group-like branches in the dependence of half-life on the α-particle energy were observed [11].

The detailed analysis shows that precise linear dependence of the logarithm of α-decay half-lives on the reciprocal of square root of the α-particle energy for even-even isotopes of the U, Pu and Cm there are (Fig.3). Also, mass numbers of these isotopes are regularly increased along the line corresponding to the Geiger Nuttall law.

At the same time for some even-even isotopes of the Po, Ra and Th such regular behaviour of the dependence of the $\ln T_{1/2}$ versus $1/\sqrt{E_\alpha}$ was broken (Fig.4).

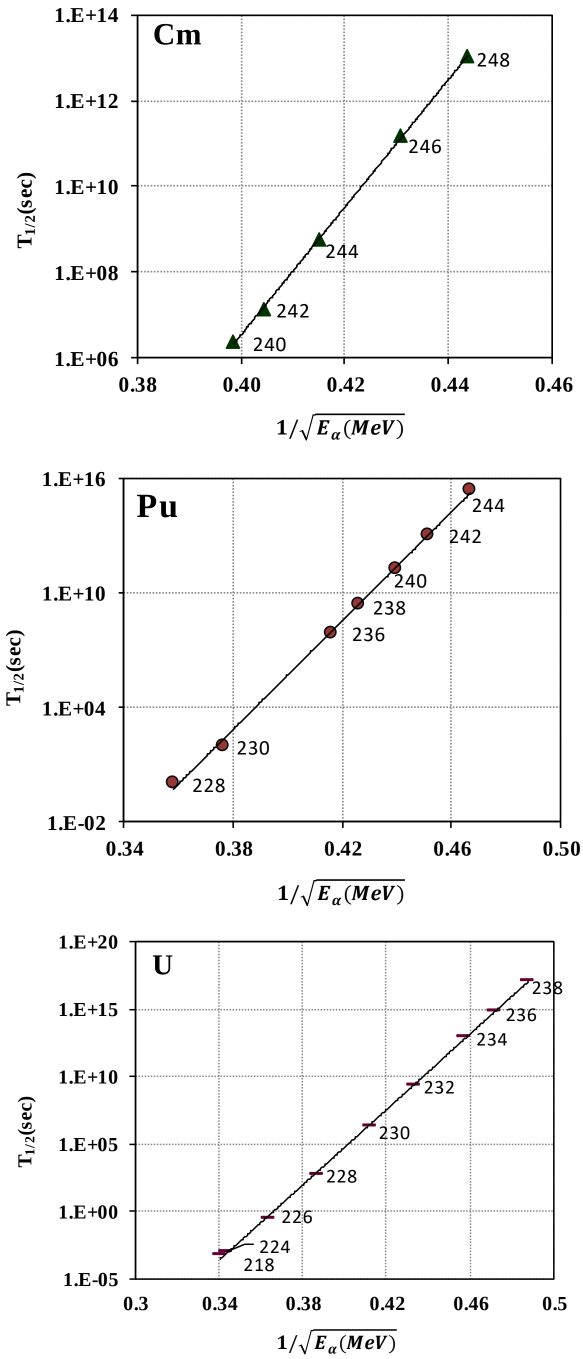

Fig. 3. The same as in Fig. 2 for isotopes of the U, Pu and Cm.

It can be seen from Fig.4 that ^{196,198,208,210}Po, ^{214}Ra and ^{216}Th are off the regular behaviour of mass numbers which are increased along the Geiger-Nuttall law line. REN Zhong-Zhou et al. [12] attempted to explain this effect by the nuclear shell model. For the isotopes of ^{210}Po, ^{214}Ra and ^{216}Th number of neutrons is, really, N=126 (magic number) and neutron shell is closed (see Fig.5).

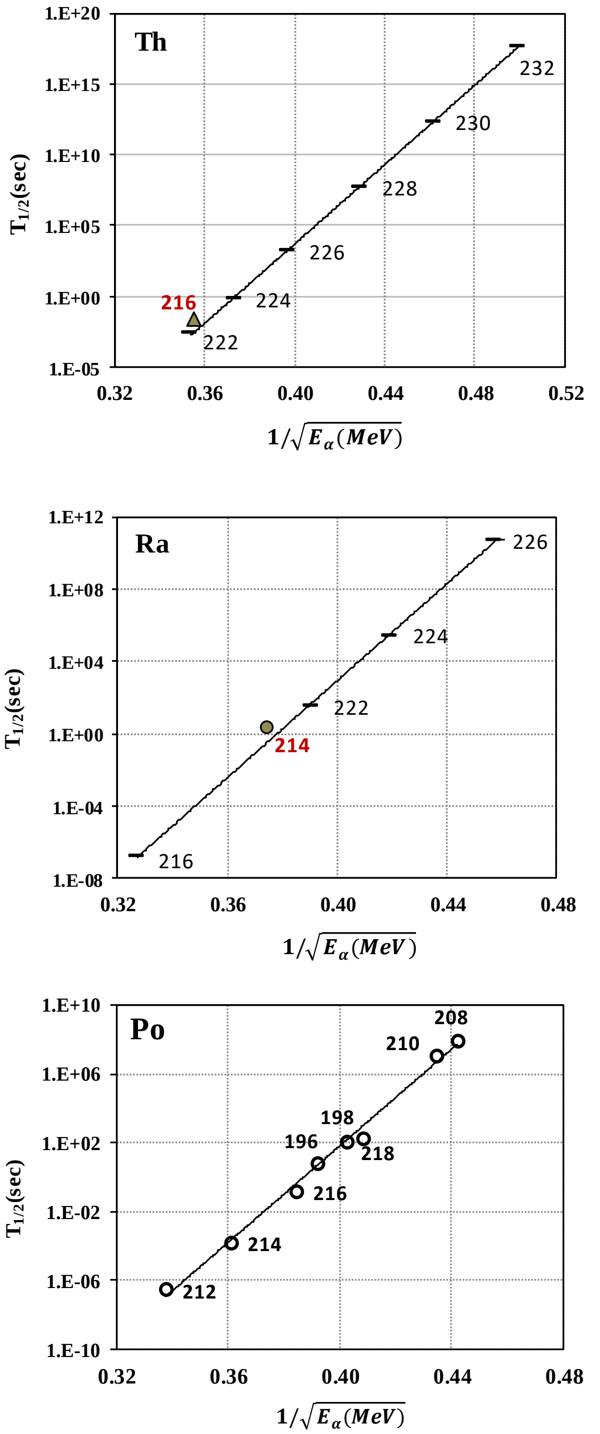

Fig.4. The same as in Fig. 2 for isotopes of the Th, Ra and Po.

Next energy levels are usually split from the closed shell by appreciable energy gap.

So, sudden breaks of the regular behaviour of mass numbers for the ^{208,210}Po, ^{214}Ra and ^{216}Th are, perhaps, caused by the gap of energy levels around magic number N=126. Similar non-regularity for the ^{196,198}Po can be explained by closing the subshell $3P_{1/2}$ (N=112). Also, steep leaps for isotopes ^{252}Cf and ^{254}Fm on the Geiger-Nuttall lines were observed (see Fig.6).

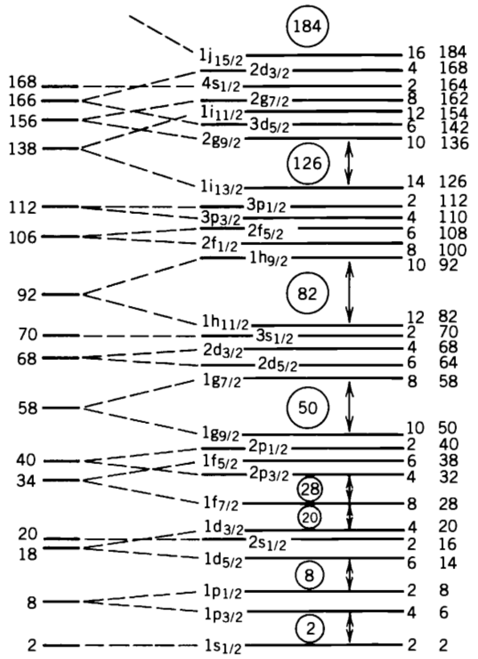

Fig. 5. Energy levels of the nuclear shell model [13].

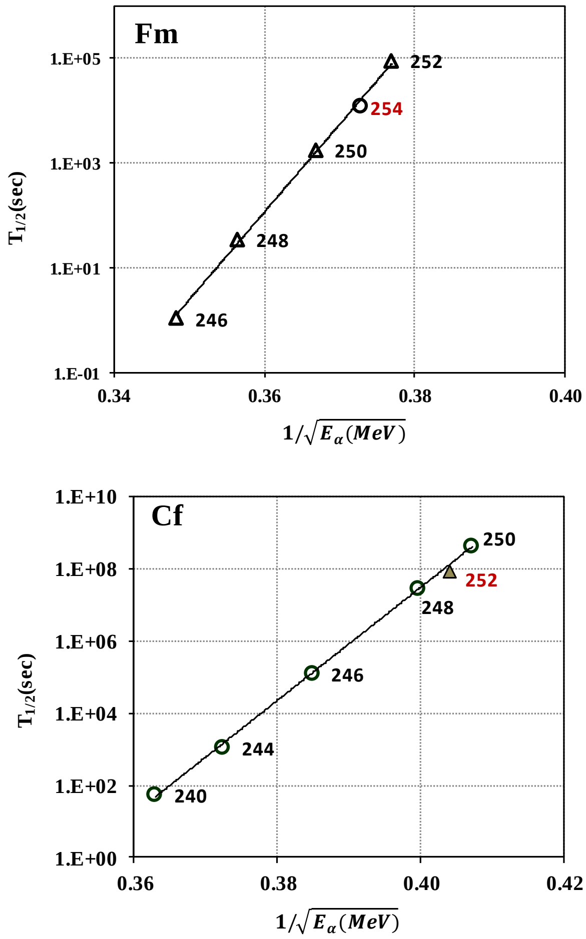

Fig. 6. The same as in Fig. 2 for isotopes of the Fm and Cf.

In these cases, the neutron number N=154 and the subshell $1i_{11/2}$ is closed. In addition, a straight relation between the $\ln(T_{1/2})$ and $1/\sqrt{E_\alpha}$ appears for isotonic chains with N=124, 126, 150 and 152 [12] (see Fig.7). However, theoretical explanation of this regularity, as far as we know, is not available.

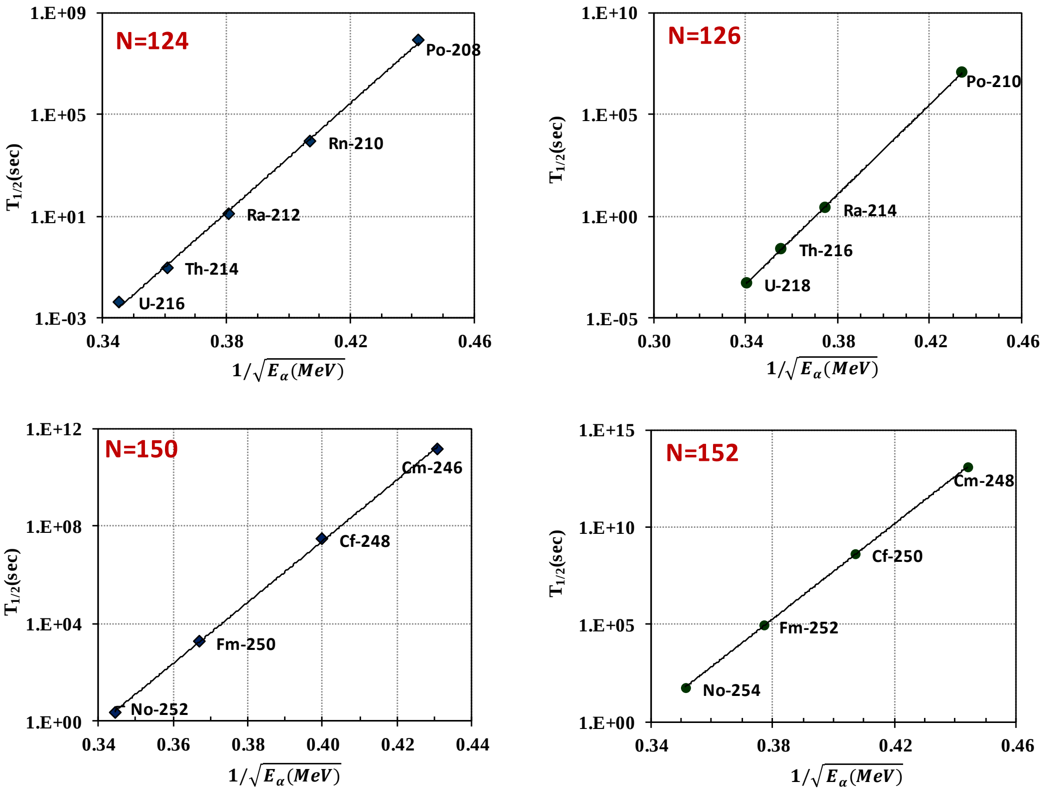

Fig. 7. The same as in Fig. 2 for isotonic chains of N=124, 126, 150 and 152.

IV. CONCLUSIONS

1. The Geiger-Nuttal law was deduced from the quantum Gamow theory. The ground-to-ground state α-transitions for natural and artificial ~450 α-active nuclides were analyzed using the Geiger-Nuttall rule. Five group-like branches for considered nuclides were observed.
2. Precise linear dependence of the logarithm of α-decay half-lives on the reciprocal of square root of the α-particle energy for even-even isotopes of the U, Pu and Cm were established. Mass numbers of the isotopes are regularly increased along the Geiger-Nuttall line.
3. For some even-even isotopes of the Po, Ra and Th regular increasing the mass number along the Geiger-Nuttall line was broken. These results were explained by the nuclear shell model.
4. A straight relation between the $\ln T_{1/2}$ and $1/\sqrt{E_\alpha}$ appears for isotonic chains with N=124, 126, 150 and 152. A theoretical substantiation of this regularity, as far as we know, is not available. So, it is interesting to investigate this effect in the future.

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