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AVERAGE (n,α) CROSS SECTION INDUCED BY SLOW NEUTRONS

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ABSTARCT

For average (n,α) cross section of slow neutrons were obtained working formulae based on the statistical model. As example, for several target nuclei with A=91−149 were calculated the transmission factors of α particle.

1. INTRODUCTION

Investigation of charged particle emission reactions induced by slow neutrons ($E_n \leq 1MeV$) is of interest for both the understanding of basic nuclear physics problems and nuclear energy applications. In particular the study of (n,α) cross section is important for research of nuclear reaction mechanisms and α-cluster structure [1], as well as for estemating radiation damage due to helium production in the structural materials of fission and fusion reactors [2]. In addition, some (n,α) cross sections are of importance to nuclear astrophysics calculations [3,4]. Because of this, it would be very useful to obtain the correct expression for (n,α) cross section . Several fomulae have been suggested to calculate average (n,γ), (n,p) and (n,α) cross sections for neutron energy up to several MeV [4]. For average (n,γ) cross section detailed consideration was perfected by Lane and Lynn [5]. However, for specific calculation of average (n,α) cross section it must be taken into account statistical fluctuations of α-widths [6]. Main features of average (n,α) cross section have been obtained in refs. [7,8] In this paper we report on the attempt of more detail consideration for average (n,α) cross section based on the statistical model of nuclear reaction.

2. AVERAGE (n,α) CROSS SECTION

2.1 RESONANCE THEORY

For slow neutrons in the energy range ΔE_i, near resonance state i, averaged (n,α) cross section can be written as

$$\left\langle \sigma^{i}_{n\alpha}(J,l) \right\rangle = \frac{\int_{\Delta E_i} \sigma_{n\alpha}(J,l,E)\Phi(E)\cdot dE}{\int_{\Delta E_i} \Phi(E)\cdot dE} \qquad (1)$$

where J is the compound nucleus total angular momentum, l is the angular

momentum of neutrons, E is the neutron energy and $\Phi(E)$ is the neutron flux. If for ΔE_i we assume $\Phi(E) \approx const$, then using the one-level Breit-Wigner formula [9] we get

$$\langle \sigma_{n,\alpha}(J,l) \rangle = \frac{\pi \bar{\lambda}^2 g(J)}{\Delta E} \sum_{i=1}^{N} \int_{-\infty}^{+\infty} \frac{\Gamma_n(J,l) \cdot \Gamma_\alpha(J,l)}{(E-E_i)^2 + \frac{\Gamma_i^2}{4}} dE \tag{2}$$

Here λ is the wavelength of the incident neutrons divided by 2π, g(J) is the statistical weight factor, ΔE is the energy interval of neutrons, N is the resonance number, Γ_n, $\Gamma\alpha$ and Γ are the neutron, alpha and total widths of i state. Using the formula

$$\int_{-\infty}^{+\infty} \frac{dx}{a^2 + x^2} = \frac{\pi}{a} \tag{3}$$

we may write

$$\langle \sigma_{n,\alpha} \rangle = 2\pi^2 \bar{\lambda}^2 \sum_l \sum_J \frac{g(J)}{D(J)} \left\langle \frac{\Gamma_n(J,l) \cdot \Gamma_\alpha(J,l)}{\Gamma(J,l)} \right\rangle \tag{4}$$

Here $D(J) = \Delta E/N$ is the mean spacing of resonances with angular momentum J.

2.2 STATISTICAL FLUCTUATIONS OF NEUTRON AND ALPHA WIDTHS

In order to use formula (4) in practice, it is necessary to calculate the neutron and alpha-widths fluctuation factor,

$$L(l) = \frac{\left\langle \frac{\Gamma_n \cdot \Gamma_\alpha}{\Gamma} \right\rangle}{\frac{\langle \Gamma_n \rangle \langle \Gamma_\alpha \rangle}{\langle \Gamma \rangle}} \tag{5}$$

which can be determined as

$$L(l) = \frac{\langle \Gamma \rangle}{\langle \Gamma_n \rangle \langle \Gamma_\alpha \rangle} \int_0^\infty \int_0^\infty \frac{\Gamma_n' \cdot F_n(\Gamma_n') \cdot d\Gamma_n' \cdot \Gamma_\alpha \cdot F_\alpha(\Gamma_\alpha) \cdot d\Gamma_\alpha}{(\Gamma_n + \Gamma\gamma + \Gamma\alpha)} \tag{6}$$

where F_n and F_α are the probability distributions for neutron and alpha widths . In most cases experimental values of Γ_α are satisfied conditions $\Gamma_\alpha \ll \Gamma_n$ and $\Gamma_\alpha \ll \Gamma\gamma$. Therefore expression (6) becomes

$$L(l) = \frac{\langle \Gamma \rangle}{\langle \Gamma_n \rangle \cdot \langle \Gamma_\alpha \rangle} \cdot \frac{\int_0^\infty \Gamma_n' F(\Gamma_n') d\Gamma_n' \cdot \int_0^\infty \Gamma_\alpha F_\alpha(\Gamma_\alpha) d\Gamma_\alpha}{(\Gamma_n + \Gamma_\gamma)} \tag{7}$$

Last integral gives

$$\int_0^\infty \Gamma_\alpha F_\alpha(\Gamma_\alpha) d\Gamma_\alpha = \langle \Gamma_\alpha \rangle \tag{8}$$

Thus, for reduced neutron widths $\Gamma_n' = \Gamma_n / \upsilon_l(E)^{1/2}$ (here υ_l is the penetrability factor for neutrons) using Porter-Thomas distribution [10], we have

$$L(\ell) = \frac{\langle \Gamma \rangle}{\langle \Gamma_n \rangle} \int_0^\infty \frac{\Gamma_n \exp\left[-\frac{\Gamma_n'}{2\langle \Gamma_n' \rangle}\right] d\Gamma_n'}{(\Gamma_n + \Gamma_\gamma)\left[2\pi \cdot \langle \Gamma_n' \rangle \cdot \Gamma_n'\right]^{1/2}} \tag{9}$$

From the expressions (4), (5) and (9) using the formula from ref [11]

$$\int_0^\infty \frac{x^2 \exp(-x^2)}{x^2 + \beta^2} dx = \frac{\sqrt{\pi}}{2} \left\{1 - \sqrt{\pi} \cdot \beta \exp(\beta^2)[1 - \Phi(\beta)]\right\} \tag{10}$$

we get finally

$$\langle \sigma_{n,\alpha} \rangle = 2\pi^2 \bar{\lambda}^2 \sum_\ell \sum_J \frac{g(J) \langle \Gamma_\alpha(J,\ell) \rangle}{D(J)} \left\{1 - \sqrt{\pi} \beta \exp(\beta^2)[1 - \Phi(\beta)]\right\} \tag{11}$$

Here $\Phi(\beta)$ is the error function

$$\Phi(\beta) = \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-t^2) dt \quad \text{and} \quad \beta = \frac{\langle \Gamma_\gamma \rangle}{2\langle \Gamma_n' \rangle \cdot \sqrt{E} \cdot \upsilon_\ell \cdot \varepsilon(J)} \tag{12}$$

where $\varepsilon(J)$ is the factor of channel spins [12]:

$$\varepsilon(J) = \begin{cases} 2 & \text{if both } |J-I| \le I \pm 1/2 \le J+1 \\ 1 & \text{if } |J-I| \le \text{ only one of } I \pm 1/2 \le J+1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

In order to calculate the average (n,α) cross section by the expression (11) it is necessary to determine average α-width $\langle \Gamma_\alpha \rangle$. According to Blatt and Weisskopf [13] average α-width is given by

$$\langle \Gamma_\alpha \rangle = \frac{D(J)}{2\pi} \cdot T_\alpha \qquad (14)$$

Here T_α is the transmission factor which is a sum of the surface reflection and penetrability through Coulomb and centrifugal barriers. In the case of "black-nucleus" approximation may be written [14]

$$T_\alpha = \exp\left\{ -2\sqrt{\frac{2\mu}{\hbar^2}} \int_{R_i}^{R_o} \left[V(r) + \frac{2Ze^2}{r} + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} - E_\alpha \right]^{1/2} dr \right\}; \qquad (15)$$

Here μ is the reduced mass of α-particle, Z is the proton number of daughter-nucleus, e is the charge of electron, l is the angular momentum of α-particle and E_α is the energy of α-particle; R_i and R_o are the turning points. Nuclear potential V(r) has been obtained by Igo [15]

$$V(r) = -1100 \exp\left[\frac{1.17 \cdot A^{1/3} - r}{0.574} \right]; \text{ MeV.} \qquad (16)$$

Thus the transmission factor T_α can be obtained by the expressions (15) and (16). As example, some results of our calculation for several nuclei are shown in Table 1 in the Appendix. Other parameters of the formulae (11) and (12) can be determined from the compilation [16].

APPENDIX

Table 1. Transmission factor for (n,α) reaction

Target Nucleus	E_α (MeV)	T_α, 10^{-6}		
		l=0	l=1	l=2
Zr-91	5.41	0.26	0.20	0.11
Mo-95	6.12	1.50	1.20	0.70
Te-123	7.34	0.22	0.18	0.12
Nd-143	9.44	3.90	3.20	2.20
Sm-147	9.84	4.40	3.70	2.50
Sm-149	9.17	0.48	0.40	0.28

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