Analysis of a virtual state using the complex scaling method

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We investigate the virtual state using a simple schematic two-body model which simulates 8Be+n system and the complex scaling method.

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INTRODUCTION

The complex scaling method (CSM) [1, 2] has been shown to be very useful for studying weak binding states strongly coupled with continuum states. However, it has been discussed to have difficulty in description of broad resonance states and virtual states which need a scaling angle near $\frac{\pi}{4}$ that is a limit due to the analyticity. In this work, we discuss that the CSM is very useful tool to study the virtual state. There is no previous demonstration (except our previous works [3, 4]) that the CSM can be applied successfully for investigation of a virtual state. Recently, applying the CSM to the $\alpha+\alpha+n$ three-body model for ^{9}Be, we have shown that the sharp peak of the photo-disintegration cross section experimentally observed just above the ^{8}Be(0^{+})+n threshold is dominantly explained as a $1/2^{+}$ virtual state of the ^{8}Be(0^{+})+$n$two-body configuration [3]. Furthermore, in the framework of the CSM, the structure of a virtual state in a s wave using a simple schematic two-body model was discussed [4].

In the present report, we calculate the virtual state energy using the continuum level density and the phase shift corresponding to the virtual state. In our previous work [4], we concluded that the virtual state has a strong influence on the scattering observables when it approaches the zero energy near the physical scattering region.

COMPLEX SCALING METHOD

In the CSM, the relative coordinate r is rotated as $\vec{r} \rightarrow \vec{r}e^{i\theta}$ in the complex coordinate plane [5]. The complex-scaled Hamiltonian H^{θ} and wave function $\Psi^{v}_{J^{\pi}}(\theta)$ are defined as $U(\theta)HU(\theta)^{1}$ and $\mathrm{U}(\theta)\Psi^{v}_{J^{\pi}}$, respectively, and see Ref. [1, 2] for details. The Schrodinger equation can be written as

$$H^{\theta}\Psi^{v}_{J^{\pi}}(\theta) = E^{\theta}_{v}\Psi^{v}_{J^{\pi}}(\theta), \qquad (1)$$

where J^{π} is the spin and parity, v is the state index, and θ is a scaling angle being a real number.

Applying the L^2 basis function method, the radial wave function is expanded as

$$\Psi^{v}_{J^{\pi}}(\theta) = \sum_{n=1}^{N} c_n^{J^{\pi}v}(\theta)\phi_n(r), \qquad (2)$$

where $\phi_n(r)$ is an appropriate basis function set. The expansion coefficients $c_n^{J^{\pi}v}$ and the complex energy eigenvalues E^{θ}_{v} are obtained by solving the complex-eigenvalue problem given in Eq. (1). The complex energies of resonant states are obtained as $E_r = E_r^{res} - i\Gamma_{\mathrm{r}}/2$, when $tan^{-1}(\Gamma_r/2E_r^{res}) < 2\theta$. The Hamiltonian is given as

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \mathrm{V(r)}. \qquad (3)$$

For simplicity, we put $\frac{\hbar^2}{2\mu} = 1$ (MeV fm^2). Where we assume the simple schematic two-body potential

$$V(r) = -V_1\exp(-ar^2), \qquad (4)$$

where $a = 0.16$ fm^{-2}. This potential was introduced in Ref. [6].

RESULTS

In our previous work [3,4] we confirmed that the virtual state is responsible for the enhancement of the photo-disintegration cross section near threshold

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in s-waves and when the virtual state approaches to the zero energy near the physical scattering region, it has a strong influence on the scattering observables. In the present work, we analyze the continuum level density and the phase shifts applying a simple schematic two-body model simulating the ^{8}Be+n system.

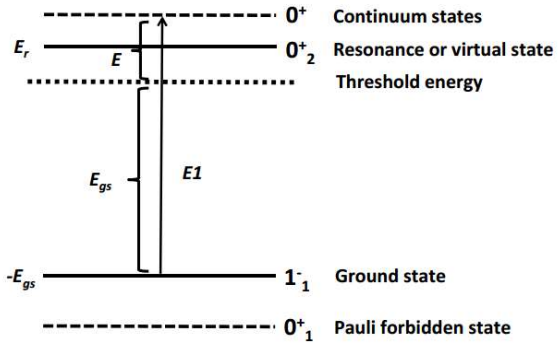

Figure 1. The energy level diagram of the two-body potential model simulating ^{9}Be. The dotted line represents the threshold energy.

In Fig. 1, we show the energy level diagram of the two-body potential model simulating ^{8}Be+n. The $J^\pi = 0^+$ and 1^- states are obtained by solving the complex-scaled Schrodinger equation. The potential strength V_1 in Eq. (4) is taken to reproduce one bound $J^\pi = 0^+$ of s waves. But this $J^\pi = 0^+$ solution is assumed to be the Pauli forbidden state, because in this model we simulate the $^8\mathrm{Be}(0^+) + n$ system which has the Pauli-forbidden $(0s)$ neutron configuration. Therefore, the 1^- solution describes the ground state.

In the CSM, the virtual state cannot be obtained as an isolated solution, but the continuum solutions are considered to include the effect of the virtual state. In order to see the resonance effect on the phase shift clearly, we calculate the phase shift subtracting the resonance term from $\delta^N(E)$ as

$$\delta^N_{N_r^\theta}(E) = \delta^N(E) - \sum_{r=1}^{N_r^\theta} \int_0^E dE' \frac{\Gamma_r/2}{(E'-E_r^{res})^2+\Gamma_r^2/4} \quad . \tag{5}$$

The Eq. (5) can be applied to calculate a virtual state contribution to phase shifts if there is a single pole obtained like a resonant pole on the first Riemann sheet. But the virtual states cannot be obtained as the isolated pole in the CSM because those are located on the second Riemann sheet covered by the rotated first Riemann sheet as we discussed in the our previous work [3]. Therefore, it is obvious that in the present work, segregated single pole from continuum states for virtual state cannot be obtained in the energy eigenvalue distribution. However, it is possible to get the information of the virtual state pole in terms of the continuum solutions in the CSM.

As can be seen from Fig.6 in Ref. [4], an additional bound state appears suddenly at $V_1 = -1.43$ MeV which can be explained following Levinson theorem

$$\delta^N_\theta(E=0) - \delta^N_\theta(E\to\infty) = N_b, \tag{6}$$

where N_b is the number of bound states. If the attraction of the potential strength is slightly increased, the virtual state becomes bound state.

When a bound state of s-wave solutions changes to a virtual pole, we call a critical point (v_c) of the coupling constant (potential strength). The continuum level densities $\Delta_\theta(v_c^+)$ and $\Delta_\theta(v_c^-)$, can be calculated for $|v_c| + \epsilon$ and $|v_c| - \epsilon$. An additional bound state appears in the former, but a virtual state pole is expected to appear in the latter. The continuum level density $\Delta_\theta(v_c^+)$ and $\Delta_\theta(v_c^-)$ are

$$\Delta_\theta(v_c^+) = \Delta_b + \Delta_c^+$$

and

$$\Delta_\theta(v_c^-) = \Delta_c^-, \tag{7}$$

where Δ_b is the continuum level density of the bound state solution. The continuum level density Δ_c^+ and Δ_c^- are calculated using the continuum solutions for $|v_c| + \epsilon$ and $|v_c| - \epsilon$, respectively. Assuming that the continuum level density $\Delta_\theta(v_c^+)$ and $\Delta_\theta(v_c^-)$ are smooth functions for the coupling constant, we can extract the continuum level density contribution from the virtual state by taking a difference between Δ_c^+ and Δ_c^-,

$$\Delta^{vir}_\theta = \Delta_c^- - \Delta_c^+. \tag{8}$$

The continuum level densities $\Delta_\theta(v_c^-)$ and $\Delta_\theta(v_c^+)$ are calculated for $V_1 = -1.42$ MeV and -1.43 MeV. The difference between Δ_c^+ and Δ_c^-, corresponding to $\Delta^{vir}_\theta(E)$ is given in Fig. 8 of Ref. [4]. From the continuum level density $\Delta^{vir}_\theta(E)$, we obtained the phase shift $\delta^{vir}(E)$ of the virtual state which is shown in Fig. 9 of Ref. [4].

Analyzing of the phase shift $\delta^{vir}(E)$ suggests a logarithmic function of E and also the continuum level density $\delta^{vir}(E)$ looks like a behavior of $1/(E - E^{vir})$. Assuming $\delta^{vir}(E) \infty 1/(E - E^{vir})$, we can extract the energy E^{vir} which corresponds to the virtual pole position on the second Riemann sheet. At energies E_i, we have the following relation

using $\Delta_\theta^{vir}(E_{i+k})$ at $E_{i+k} = E_i + kh$, where h is an energy step:

$$\frac{\Delta_\theta^{vir}(E_{i+k})}{\Delta_\theta^{vir}(E_i)} = \frac{E_i - E_{ik}^{vir}}{E_{i+k} - E_{ik}^{vir}}, \qquad (9)$$

where E_{ik}^{vir} is the virtual pole energy obtained by using $\Delta_\theta^{vir}(E_{i+k})$ $(k = 0, 1, 2, \dots, n_i)$ at E_i. Solving Eq. (9), we obtain

$$E_{ik}^{vir} = \frac{kh}{\Delta_\theta^{vir}(E_i)/\Delta_\theta^{vir}(E_{i+k}) - 1} - E_i. \qquad (10)$$

From the continuum level density $\Delta_\theta^{vir}(E)$ shown in Fig. 8 of Ref. [4] and taking $h = 0.001$ and $n_i = 5$, we can calculate E_{ik}^{vir} and show the results in Table 1.

The present results of virtual state E_{ik}^{vir} is comparable with the result obtained by Jost function method [7]. The Jost function method can be easily applied to the present two-body model and we obtained a solution of the virtual state at $E^{vir} \approx -4.97\text{x}10^{-6}$ MeV.

Table 1. The virtual pole energy E_{ik}^{vir} calculated from the continuum level density $\Delta_\theta^{vir}(E)$.

i	E_{i1}^{vir}	E_{i2}^{vir}	E_{i3}^{vir}	E_{i4}^{vir}	E_{i5}^{vir}
2	-0.00045	-0.00052	-0.00055	-0.00061	-0.00062
3	-0.00079	-0.00085	-0.00100	-0.00101	-0.00110
4	-0.00099	-0.00131	-0.00129	-0.00145	-0.00153
5	-0.00182	-0.00162	-0.00184	-0.00194	-0.00192
6	-0.00119	-0.00186	-0.00205	-0.00199	-0.00213
7	-0.00263	-0.00265	-0.00246	-0.00260	-0.00284
8	-0.00268	-0.00228	-0.00257	-0.00296	-0.00309
9	-0.00160	-0.00247	-0.00313	-0.00330	-0.00322
10	-0.00338	-0.00398	-0.00401	-0.00380	-0.00371
11	-0.00465	-0.00443	-0.00403	-0.00386	-0.00403
12	-0.00411	-0.00347	-0.00333	-0.00370	-0.00421
13	-0.00249	-0.00269	-0.00347	-0.00425	-0.00476
14	-0.00291	-0.00403	-0.00494	-0.00545	-0.00561

Comparing the results calculated both method, the present result calculated by the CSM can be considered reasonable because in the CSM complex eigenvalue problems have to be solved with the basis expansion method. Therefore, to keep a numerical accuracy with high digits is difficult for the CSM.

SUMMARY

We employed the simple schematic two-body model and the CSM for investigation of the virtual state. The virtual state energy calculated using the continuum level density and the phase shift of the virtual state. The calculated results of the virtual state solutions in the CSM and the Jost function methods are comparable.

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