Quantum gravitational force and its consequences

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We introduce a new quantum gravitational force

$$F_{Pl} = G_N \frac{\hbar}{c}\frac{m}{r^3}$$

by using the fundamental constants, like $G_N, \hbar, c$. where $G_N, \hbar$ and c are the Newtonian, the Planck constants and velocity of light. It turns out that conditions of equalities of this force with the Newtonian, the strong interacting-Yukawa type forces give exact physical meaning of the Planck length and the Compton length of a wave. Moreover, equality conditions of this force with the Coulomb and the Dirac magnetic monopole's forces give rise to introduce the concept of the running coupling constant

$$\alpha(E) = \alpha \ln(1 + \frac{E}{\mu})$$

and to obtain mass formula $M_{mon} = M_{Pl}\frac{1}{\sqrt{\alpha}}$ for a magnetic monopole, where $\alpha = \frac{e^2}{\hbar c}$ is the structure constant. Physical meaning of the vacuum energy is also considered.

Key words: Quantum gravitational force, Planck and Compton lengths, Coulomb law, Yukawa and Newtonian potentials, Magnetic monopole, vacuum energy.

1. INTRODUCTION OF A NEW FORCE

Let us introduce a new force by analogy with obtaining formal formula of the Planck length with using fundamental physical constants: $G_N, \hbar, c$. That is

$$F_{Pl} = G_N \frac{\hbar}{c}\frac{m}{r^3} \qquad (1)$$

We call it a quantum gravitational force or the Planck-Newtonian force for mass value m. For example, quantity of this force between an electron and a proton which are located at the distance $r_0 = 10^{-10}$m is

$$F_{PN}^{ep} = G_N \frac{\hbar}{c}\frac{\sqrt{m_e}\sqrt{m_p}}{r_0^3} = 9.16 \times 10^{-52} H \qquad (2)$$

Moreover, a force requiring for the Big Bang of the Universe is

$$F_{PN} = G_N \frac{\hbar}{c}\frac{M_U}{l_{Pl}^3} = 1.67 \times 10^{104} H \qquad (3)$$

where $M_U = 3 \times 10^{52}$ kg and $l_{Pl} = \sqrt{\frac{G_N \hbar}{c^3}}$ are the total mass of the Universe and the Planck length.

2. CONDITION OF EQUALITY BETWEEN THE NEWTONIAN AND QUANTUM GRAVITATIONAL FORCES

Let us propose the following identity for any two identitical particles with mass m:

$$G_N \frac{m^2}{r^2} = G_N \frac{\hbar}{c}\frac{m}{r^3} \qquad (4)$$

which immediately gives a physical foundation of the Compton length of a wave:

$$r = \lambda = \frac{\hbar}{mc}. \qquad (5)$$

This formula with the Einstein famous formula $E = mc^2$ gives wave nature of particles:

$$\lambda = \frac{\hbar}{mc^2} = \frac{\hbar c}{E}$$

and therefore

$$\lambda = \frac{\hbar}{mc}, \quad E = \hbar\omega, \quad \omega = \frac{c}{\lambda} \qquad (6)$$

as it should be.

3. CONDITION OF EQUALITY BETWEEN STRONG AND THE PLANCK-NEWTONIAN FORCES

The strong force is given by the Yukawa potential force

$$F_Y = \frac{gmc^2}{r} e^{-mr} \qquad (7)$$

where $g \approx 1, e^{-mr} \approx 1$ for short distances. Thus the condition

$$\frac{mc^2}{r} = G_N \frac{\hbar}{c}\frac{m}{r^3} \qquad (8)$$

gives the Planck length:

$$r = L_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \qquad (9)$$

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and therefore we can obtain exact physical foundation: what is the Planck length in the physical meaning. It means that unification of the strong interaction force and the quantum gravitational force takes place in a domain characterized by this Planck length.

4. CONDITION OF EQUALITY OF THE COULOMB AND THE PLANCK-NEWTONIAN FORCES

The equality for two electric charged particles with charge q and mass m:

$$k_e \frac{q^2}{r^2} = \frac{G_N \hbar}{c} \frac{m_q}{r^3} \qquad (10)$$

where

$$k_e = 8.99 \times 10^9 \frac{H \cdot m^2}{C},$$

$$q = 1.602 \times 10^{-19} C$$

gives for the electron with a genuine kernel

$$r_e = l_e = \frac{G\hbar}{c} \frac{1}{k_e} \frac{m_e}{q^2} = 9.268 \times 10^{-56} m, \qquad (11)$$

and for the proton

$$r_p = l_p = 1.702 \times 10^{-52} m, \qquad (12)$$

where the quantum number

$$\Lambda = \frac{k_e q^2 \cdot c}{G_N \hbar} = 9.83 \times 10^{24} \frac{kg}{m} \qquad (13)$$

is the same for any particles with mass m_i. Here, we can also formally write formulas (11) and (12) in the language of energy values:

$$E_e = \frac{\hbar c}{l_e} = 3.411 \times 10^{29} J, \qquad (14)$$

$$E_p = \frac{\hbar c}{l_p} = 1.858 \times 10^{26} J, \qquad (15)$$

It means that unification of the Coulomb and quantum gravitational forces takes place at very small distances or extremally high energies. In order to unify the Coulomb force with strong and quantum gravitational forces (8) given by the Planck energy scale

$$E_{Pl} = \frac{\hbar c}{L_{Pl}} = 1.956 \times 10^9 J, \qquad (16)$$

it is necessary to introduce the concept of the running coupling constant $\alpha(E) = \alpha \ln\left(1 + \frac{E}{\mu}\right)$ in the formula (10). The result reads for the proton

$$\ln\left(1 + \frac{E_p}{\mu}\right) \times 1.858 \times 10^{26} J = 1.956 \times 10^9 J.$$

From which, we have

$$\ln\left(1 + \frac{E_p}{\mu}\right) = 1.053 \times 10^{-17},$$

$$1 + \frac{E_p}{\mu} = \exp[1.053 \times 10^{-17}] = 1 + \delta$$

and therefore

$$\alpha(E) = \alpha \ln\left(1 + \frac{E}{\mu}\right) \approx \alpha \ln(1 + \delta),$$

$$\frac{E_p}{\mu} \sim \delta = 1.053 \times 10^{-17}.$$

5. THE MAGNETIC MONOPOLE

If we introduce a magnetic monopole with a magnetic charge g, then an electric charge e is exactly quantized by the Dirac formula

$$g \cdot \frac{2e}{\hbar c} = N, \qquad (17)$$

where N is an integer number. Let $N = 2$, then a magnetic field of the monopole is given by the formula

$$\vec{B} = -\frac{\hbar c}{e} \frac{\vec{r}}{r^3}. \qquad (18)$$

An another magnetic monopole with the magnetic charge $\hbar c/e$ interacts with this field by the formula

$$F_m = \left(\frac{\hbar c}{e}\right)^2 \frac{[\vec{r} \times \frac{\vec{v}}{c}]}{r^3}, \qquad (19)$$

where $\vec{v}$ is a velocity of the incoming monopole. Let this monopole moves with the velocity of light in a direction perpendicular to the field (18). Then we have an equality

$$\frac{\hbar c}{(e^2/\hbar c)} \frac{1}{r^2} = \frac{G_N \hbar}{c} \frac{M_{mon}}{r^3}, \qquad (20)$$

where $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.03599}$ and we assume $r = \frac{\hbar}{M_{mon} c}$. Then we have very nice formula for a mass of the magnetic monopole by using four fundamental constants $G_N, \hbar, c, \alpha$:

$$M_{mon}^2 = \frac{\hbar c}{G_N}(137.03599) = M_{Pl}^2 \frac{1}{\alpha} \qquad (21)$$

or

$$M = 2.5481 \times 10^{-7} kg =$$

$$= 1.4291 \times 10^{17} \frac{TeV}{c^2}, \qquad (22)$$

Here $1 kg = \frac{1}{1.783} \times 10^{36} \frac{eV}{c^2}, M_{Pl} = 2.177 \times 10^{-5} g$ is the Planck mass. This beautiful formula (21) is also arisen from the classical physics:

$$\frac{\hbar c}{(e^2/\hbar c)} \frac{1}{r^2} = G_N \frac{M_{mon}^2}{r^2}. \qquad (23)$$

It means that the magnetic monopole with mass M_{mon} maybe played a role as a limit of a classical particle which possesses simultaneously wave-mechanical properties with the Compton length $\lambda_{mon} = \frac{\hbar}{M_{mon} c}$, if it exists in nature.

6. THE QUANTUM GRAVITATIONAL FORCE AND THE VACUUM ENERGY

We assume that the Casimir effect or the vacuum energy between two plates is given by the formula

$$\frac{1}{6}\frac{\hbar c\pi^2}{240a^4}A = \frac{m_{Pl}}{a^3}\frac{G_N\hbar}{c} \qquad (24)$$

or

$$a\cdot\rho_{Pl} = \frac{1}{6}\frac{c^2}{240}\frac{\pi^2}{G_N} = 9.23\times10^{24}\frac{kg}{m}, \qquad (24)$$

where a is the distance between the two plates, coefficient $1/6$ is due to per unit degrees of freedom determined by three Euler angles and three unit vectors in the coordinate system of coordinates and $\rho_{Pl} = m_{Pl}/A$ is the density of the plate mass per unit area. We see that quantity (25) almost coincides with quantum number

$\Lambda = 9.83\times10^{24}\frac{kg}{m}$ given by the formula (13).

It means that the quantum gravitational force (1) has a universal character for different physical processes in the microworld.

Our final remark is that we know that the Newtonian potential $\Phi_N = -G_N m/r$ is played a vital role in the classical Einstein theory of gravitation, where in the weak field limit there exists connection:

$$g_{00}(x) = \eta_{00} + H_{00}(x), \quad H_{00}(x) = -2\Phi_N,$$

η_{00} is the Minkowski metric. Then for the quantum gravitational potential$\Phi_{PN} = -G_N\hbar/cr^2$, we have

$$\widehat{g_{00}}(x,\hbar) = \eta_{00} + \widehat{H_{00}}(x,\hbar),$$

where

$$\widehat{H_{00}}(x,\hbar) = -2\Phi_{PN}.$$

Therefore, the quantum gravitational force (1) may be useful for the quantization of Einstein's general theory of gravitation, where the Planck constant $\hbar$ enters exactly.

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