Scattering cross sections in complex scaling method

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I. INTRODUCTION

The studies of scattering problems in the nuclear science have been developed by using various experimental techniques and theoretical methods so far. Recently, with the development of unstable nuclear beam experiments, much interest has been concentrated on many-body resonances. As a very promising method, the complex scaling method (CSM) [1] has been applied to those resonance problems. This approach seems to be very promising to unify the description of the nuclear structure, and reactions and also in nuclear data evaluations, especially, for light nuclear mass systems [2].

In this work, we study the scattering phase shifts using CSM. The scattering phase shifts have been shown to be calculated from the continuum level density (CLD) [3]. We develop the method of calculating CLD to investigate the effects of the resonant states which is related to the nuclear structures, separated from continuum states providing background contributions in the phase shifts. This new method is applied to the complex scaled orthogonality condition model [4] of several scattering systems including the n-α and α-α systems. The background phase shift is also obtained by using the residual continuum solutions in CSM. We discuss the problems of scattering in this framework, and show that this method is very useful in the investigation of the effect of the resonance in the observed scattering cross sections.

II. EVALUATION AND DISCUSSION

The CLD $\Delta(E)$ is given as

$$\Delta(E) = -\frac{1}{\pi}\,\mathrm{Im}\left\{Tr\left[G(E) - G_0(E)\right]\right\}, \tag{1}$$

where $G(E) = (E-H)^{-1}$ and $G_0(E) = (E-H_0)^{-1}$ are the full and free Green's functions, respectively. In this study, the Hamiltonian H and H_0 are transformed by using the CSM.

The CLD is related to the scattering phase shifts δ and it can be expressed by following form in the single channel case [5]:

$$\Delta(E) = \frac{1}{\pi}\frac{d\delta(E)}{dE}. \tag{2}$$

Using this relation, we can obtain the phase shift as a function of the eigenvalues in the complex scaled Hamiltonian by integrating the CLD.

When we expand the wave functions in terms of the finite number N of the basis states, the discretized eigen states are obtained with number N and the level density can be approximated [3]

$$\Delta(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[\sum_{B=1}^{N_B}\frac{1}{E+i0-E_B} + \sum_{r=1}^{N_r^\theta}\frac{1}{E-E_r^{res}+i\Gamma_r/2} + \sum_{c=1}^{N_c^\theta}\frac{1}{E-\varepsilon_c^r+i\varepsilon_c^i} - \sum_{k=1}^{N}\frac{1}{E-\varepsilon_k^{0r}+i\varepsilon_k^{0i}}\right] \tag{3}$$

where $N = N_B + N_r^\theta + N_c^\theta$ for bound (B), resonant (r) and continuum (c) solutions. Then we can obtain the phase shift

$$\delta_N^\theta(E) = N_b\pi + \sum_{r=1}^{N_r^\theta}\left\{-\cot^{-1}\left(\frac{E-E_r^{res}}{\Gamma_r/2}\right)\right\} + \sum_{c=1}^{N_c^\theta}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_c^r}{\varepsilon_c^i}\right)\right\} - \sum_{k=1}^{N}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_k^{0r}}{\varepsilon_k^{0i}}\right)\right\}, \tag{4}$$

where $E > 0$. When we define δ_r, δ_c and δ_k as

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$$\cot\delta_r = \frac{E_r^{res} - E}{\Gamma_r/2},\ \cot\delta_c = \frac{\varepsilon_c^r - E}{\varepsilon_c^i},$$

$$\cot\delta_k = \frac{\varepsilon_k^{0r} - E}{\varepsilon_k^{0i}} \quad (5)$$

respectively, we can write the phase shift

$$\delta_N^\theta(E) = N_b\pi + \sum_{r=1}^{N_r^\theta}\delta_r + \sum_{c=1}^{N_c^\theta}\delta_c - \sum_{k=1}^{N}\delta_k. \quad (6)$$

The geometrical indications for δ_r, δ_c and δ_k are given for two energy cases, larger or smaller than real parts of eigenenergies E_r, ε_c and ε_k, in Fig.1. The phase shift δ_r for the resonances is the angle of the r-th resonant pole measured at the energy E on the real energy axis. At $E = E_r^{res}$, we have $\delta_r = \pi/2$ for every resonant pole. Furthermore, while $\delta_r > 0$ at $E = 0$, $\delta_r = \pi$ at $E = +\infty$. On the other hand, it is seen that the phase shifts from the continuum terms of the asymptotic and full Hamiltonian are lay on the 2θ line.

The cross section is described by using these phase shifts, and we can identify the contributions from every resonant pole and continuum terms. When we concentrate our interest on the contribution from a single resonant pole and other terms which are mainly described as a background phase shift, we can have the same discussion as done by Fano [6].

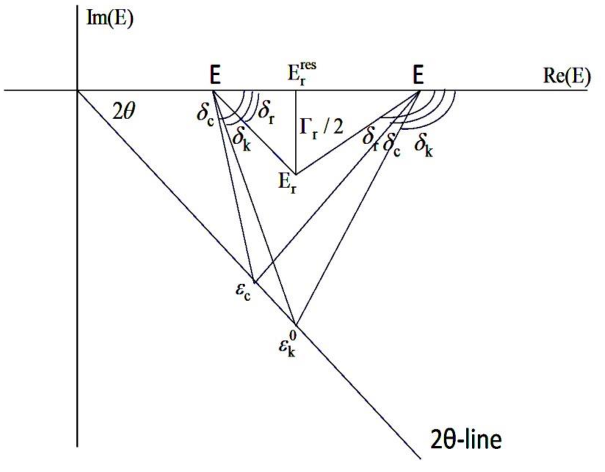

Fig. 1. The geometrical indications for phase shifts: δ_r, δ_c and δ_k.

The total and partial reaction cross sections can be calculated by using the results of phase shifts decomposed into the contributions of resonance and continuum. From the results, we can investigate the contributions of resonance and continuum states in the cross sections.

The partial cross sections σ_l for the each partial wave with index l is expressed as

$$\sigma_l = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l. \quad (7)$$

where $k^2 = \frac{2E\mu}{\hbar^2}$ and μ is the reduced mass of a system. The total cross section is expressed as

$$\sigma = \sum_l^\infty \sigma_l. \quad (8)$$

In this work, the scattering phase shifts of two-body two systems is calculated by Eq. (4) which is derived from the CLD with the extended completeness relation. After the calculation of the decomposed scattering phase shifts, the partial cross sections of low-lying states are studied with the resonance and continuum contributions for the n-α and α-α systems, respectively.

The total cross sections of the n-α system shown in Fig. 2 are calculated in terms of the scattering phase shifts by using Eq. (8). The theoretical total cross section is in reasonable agreement with the experimental data. The present results of the scattering total cross section that is performed by theoretical formulation are displayed as the dotted line. The open circles in Fig.2 show the experimental data which were taken from Ref. [7]. The partial cross sections, including contributions of the resonance and continuum terms, are given in Fig. 2 for j=1/2$^-$ and 3/2$^-$ waves of the n-α system, respectively. The dashed-, solid- and dotted-lines represent the partial cross section and the contributions of resonance and continuum terms, respectively.

The total cross section is given by a sum of the partial ones which are expressed as an interference of resonance and continuum contribution as discussed by Fano [6] due to the relation $\delta_\ell = \delta_r + \delta_c$ in the phase shifts given in Eq. (6). The continuum contributions of the cross section are almost the same behavior in both states of $p_{3/2}$ and $p_{1/2}$ as shown in Fig. 3. These contributions change the form of the cross section from a symmetric Breit-Wigner shape to asymmetric peaks. Although the resonant peak of the cross section can be clearly seen in the case of $p_{3/2}$, the $p_{1/2}$ results show a mild bump in the partial cross section, the peak of which is smaller than the peak in the resonance contribution. These interference of resonance and continuum terms in the cross section are well understood from the Fano formula.

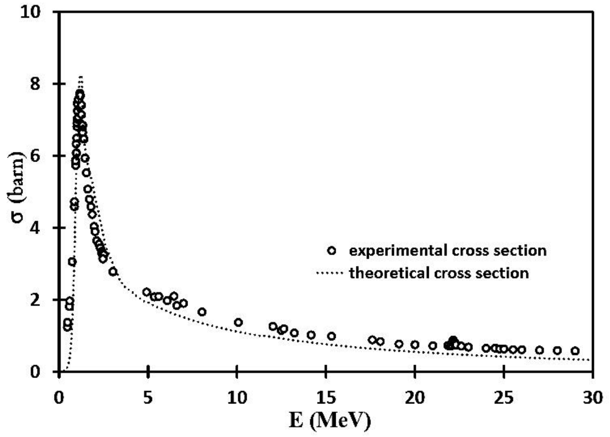

Fig. 2. Total cross section of the n-α system. The open circles display the experimental data taken from Ref. [7], and dotted lines show the present results.

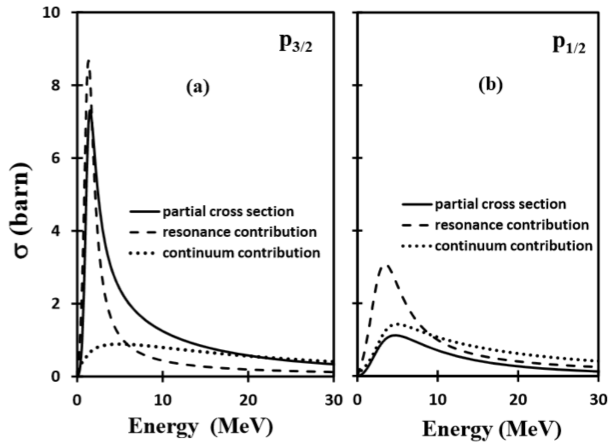

Fig. 3. The results of partial cross sections (dashed-line) and contributions from the resonance (solid line) and continuum (dotted line) terms in j=1/2- and 3/2- waves of the n-α system.

Fig. 4 shows the results of cross sections of the α-α system in the relative angular momentum L = 2, 4, 8 and 10 waves, respectively.

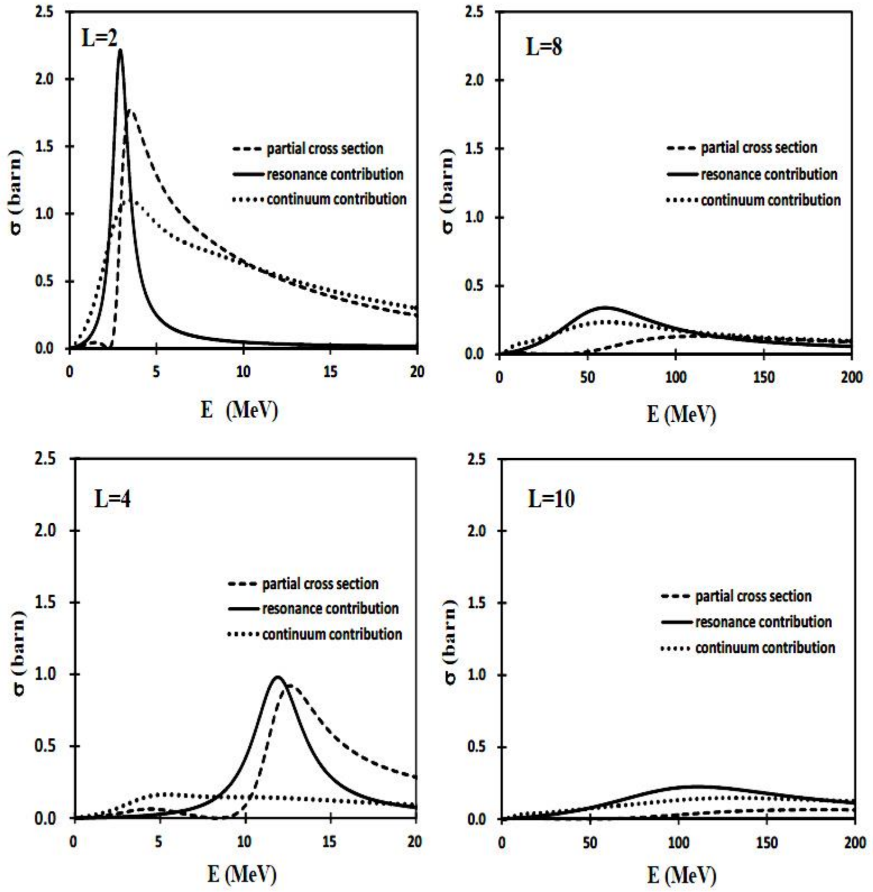

Fig. 4. The partial cross sections in L = 2, 4, 8 and 10 waves of the α-α system. The curves, dotted-lines and dashed-lines denote the results of resonance, continuum contributions of cross sections and partial cross sections, respectively.

It is interesting to see the behavior of cross sections in the relative angular momentum. From Fig. 4, it can be clearly seen in the L = 2 and 4 partial waves give a bell-shaped structure of cross section. However, a bell-shaped structure of cross sections in the L = 8 and 10 partial waves is not observed.

SUMMARY

We have investigated the scattering cross sections of the various potential systems. In the present study, not only the scattering cross sections but also the contributions of the resonance and continuum states were computed for partial states of two various systems.

ACKNOWLEDGEMENT

The authors are grateful for the support by “'R&D' Platform Formation of Nuclear Reaction Data in Asian Countries (2010-2013)”, Asia-Africa Science Platform Program, Japan Society for the Promotion of Science (JSPS).

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