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Kerr nonlinear couplers

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Directional nonlinear couplers composed of two Kerr nonlinear waveguides are studied. Quantum consistent solutions of the equations of motion are obtained showing the collapses and revivals of the field mode amplitudes. The approximate method is also adopted for the contradirectional geometry and the switching accompanied by collapses of the modes is revealed. Kerr couplers with varying linear coefficients are examined and possible controls of the switching of modes, for instance, digital switching and fast switching are demonstrated. The two-mode squeezing of vacuum fluctuations and the generation of pure states are shown.

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I. INTRODUCTION

The control of light which is propagating through two adjacent waveguides and exchanging the energy of their modes due to the evanescent field overlap, has been extensively investigated with respect to the presumable applications in optics communications, such as integrated optics devices, during the past two decades [1]. Recently nonlinear couplers composed of linear and nonlinear waveguides have been examined [2] and quantum statistical properties of such devices investigated [3]. Also nematic liquid crystals can be adopted to be used in nonlinear couplers with remarkable properties [4] and Schrödinger–cat states can be transmitted through such nonlinear couplers [5]. A similar device exhibiting interesting switching properties was also discussed in [6] based on nondegenerate optical parametric process of frequency down conversion. Entangled superpositions of macroscopically distinguishable states in a nonlinear directional coupler composed of the second and third order nonlinear waveguides can be generated [7]. Two-mode squeezing in Raman couplers was also demonstrated [8]. The quantum generalization [9] of classical equations of motion and the convenience of the use of the space dependent momentum operator instead of time dependent Hamiltonian [10] enable us to investigate the nonclassical photon statistics [11] even for the contradirectional geometry [1]. An input–output formulation of the contradirectional coupler has been developed [10, 12, 13]. Using iterations in the interaction distance or a strong stimulating field in the second harmonic mode of the asymmetric nonlinear contradirectional coupler, in general with the phase mismatch [14], an effective transfer of nonclassical properties of light from nonlinear to linear waveguides was demonstrated and it was shown that the contrapropagating geometry provides better conditions for squeezing in single modes of the symmetric optical parametric nonlinear coupler [15]. The possibility of codirectional simulations of some contradirectional couplers was also examined [16]. The nonlinear directional coupler that consists of two parallel waveguides with optical Kerr media is attractive due to intensity–dependent transmission characteristics. A codirectional configuration of a Kerr nonlinear coupler (KNC) was firstly introduced by Jensen [17]. The corresponding quantum treatment (for a review of quantum statistical properties of nonlinear couplers, see [18]) of the classical KNC gives rise to nonclassical effects, such as collapses and revivals [19] of the mode amplitudes, squeezing of quantum fluctuations and photon antibunching [20]. Quantum phase properties of the KNC were also studied [21] (for a general study of quantum phase, see [22]). Since the modification of the switching by nonlinear interactions in conventional KNC may fail for smaller input intensities, a remaining control to yield desired switching characteristics would be the coupler length only. Recently, there has been examined a bent KNC [23] (and references therein) in the framework of fabrication tolerances in contrast to the stringent parallel waveguides, in order to study an external control by variations of linear coupling coefficients [25]. The quantum treatment of the classical bent KNC gives rise to the possibility to control the switching characteristics and principal squeezing effect by adjusting the form of coupling function [24] in the same manner as shown by Hatami Hanza et al. [25]. In the previous works [20, 26], we studied codirectional and contradirectional geometry of a conventional KNC and found that, especially, contradirectional geometry permitted to get a greatly stable switching in contrary to sensitive periodical exchanges of the modes for codirectional KNC. Using obtained quantum consistent solutions we have demonstrated the collapse during the switching accompanied by enhanced squeezed quantum noise.

In the present paper, codirectional and contradirectional nonlinear couplers are discussed by means of a unified method and, in particular, their switching and quantum properties are examined, which are modified by variations of linear coupling coefficients. In the following section we begin with the Heisenberg system of equations composed for field mode operators and find their solutions for both codirectional and contradirectional configurations. In the last section we conclude main results obtained in the paper.

II. EQUATIONS OF MOTION AND THEIR SOLUTIONS

In the most of quantum mechanical theories of the light interacting with one or two atoms, temporal differential equations of motion using the corresponding Hamiltonian which describes energy of the system are adequately describing the essentials of microscopic processes. The complete quantum mechanical description of light propagation in the macroscopic system or material medium requires basic consideration of the variation of the electromagnetic field along at least two dimensions: time and spatial coordinate. Therefore, the role of the spatial progression of the field requires the evidence of using spatial differential equations. In contrast to the rigorous theory of quantum optics [27], where the material system is introduced in the form of point charges (or atoms) interacting with the field, Abram and Cohen [28] have presented a direct space formulation of the theory of quantum optics, starting with the canonical quantization procedure for the electromagnetic field inside an effective (linear and nonlinear) medium. Although, the medium is introduced through its induced polarization, or equivalently through its effective optical susceptibility, it still permits a quantum mechanical description of the field, in the sense that it can treat all problems associated with the noncommutativity of the field operators. This quasi phenomenological treatment of the material medium does separate the problem of propagation of the field from the microscopic description of the field matter interaction. A direct space formulation can be achieved by interchanging the use of Hamiltonian which has no directionality with the use of the momentum operator of the electromagnetic field $\hat{G}_z$ [28]-[31], since by definition

$$\frac{d}{dz}\hat{A}_z = \frac{i}{\hbar}\left[\hat{A}_z, \hat{G}_z\right], \qquad (1)$$

where $\hat{A}_z$ is any operator. Using this model, several propagating systems can be analyzed: a quantum mechanical space dependent amplifier, codirectional and contradirectional couplers and distributed feedback lasers [10]. Particularly, a number of stimulating papers has been devoted to directional couplers which are in close connection with the quantum theory of light propagation. A standard quantum description of directional couplers is based on, so called, a coupled mode theory (see, for instance, the work [10]). The spatial (z-dependent) Heisenberg equations involving the momentum operator, which is written by using the field mode operators for the particular system, are the foundations of the coupled mode theory. The solution of the equation of motion determines the states at all points inside the volume of the coupler, including output states at the length, say, L_{max}, where L_{max} stands for the length of the coupler. Quantum self consistency in the sense that the mode operators conserve the boson commutation relations at every point inside the coupler volume, is correctly described in the case of codirectional couplers, where two modes propagate in the forward direction. However, this quantum self-consistency of the operators involved in the Heisenberg equations of motion, does completely fail in the case of contradirectional couplers where one of the modes propagates in the backward direction. But it can be correct for the input and output operators which are defined at the points $z = 0$ and $z = L_{max}$ [13]. Therefore, one can say that the self-consistency can be unequivocally formulated for the input and output operators for both linear and nonlinear couplers regardless the fields are forward or backward propagating.

A. Codirectional nonlinear couplers

Let us begin to study in detail the case of the codirectional KNC for which two input (output) ports are specified on the right (left) handside of the nonlinear waveguides and the corresponding momentum operator in the interaction picture is given by [20]

$$\hat{G}_z = \hbar g \hat{a}_{1_z}^{\dagger 2}\hat{a}_{1_z}^2 + \hbar g \hat{a}_{2_z}^{\dagger 2}\hat{a}_{2_z}^2 + \hbar \bar{g}\hat{a}_{1_z}^{\dagger}\hat{a}_{1_z}\hat{a}_{2_z}^{\dagger}\hat{a}_{2_z} + \left(\hbar\kappa(z)\hat{a}_{1_z}\hat{a}_{2_z}^{\dagger} + \text{h.c.}\right), \qquad (2)$$

where the momentum operator $\hat{G}_z$ determines the system constructed by means of the field mode operators $\hat{a}_{1_z,2_z}$ and g, $\bar{g}$ are nonlinear coupling coefficients proportional to the third-order susceptibility. A linear coupling coefficient $\kappa(z)$ is, in general, dependent on z. The ratio $\gamma = \frac{\bar{g}}{2g}$ of effective self and cross-nonlinear coupling constants g and $\bar{g}$, respectively, can be either positive or negative depending upon the tensor nature of the third order susceptibility [32] of the nonlinear medium. We assume that a nonlinearity is always much small, therefore, these nonlinear coefficients can be taken as constants. Alternatively [10, 30], the equations of motion for the

KNC can be derived from (1), thus, giving [20]

$$\frac{d\hat{a}_{1_z}}{dz} = i\kappa(z)\hat{a}_{2_z} + 2ig(\hat{a}^\dagger_{1_z}\hat{a}_{1_z})\hat{a}_{1_z} + i\bar{g}(\hat{a}^\dagger_{2_z}\hat{a}_{2_z})\hat{a}_{1_z},$$
$$\frac{d\hat{a}_{2_z}}{dz} = i\kappa(z)\hat{a}_{1_z} + 2ig(\hat{a}^\dagger_{2_z}\hat{a}_{2_z})\hat{a}_{2_z} + i\bar{g}(\hat{a}^\dagger_{1_z}\hat{a}_{1_z})\hat{a}_{2_z}, \quad (3)$$

where $\hat{N}_z = \hat{a}^\dagger_{1_z}\hat{a}_{1_z} + \hat{a}^\dagger_{2_z}\hat{a}_{2_z}$ =constant. We substitute new bosonic operators $\hat{b}_{1_z,2_z} = \frac{1}{\sqrt{2}}(\hat{a}_{1_z} \pm \hat{a}_{2_z})$, and arrive at

$$\frac{d\hat{b}_{1_z}}{dz} = i\kappa(z)\hat{b}_{1_z} + ig\hat{M}_{1_z}\hat{b}_{1_z} + \hat{f}_{2_z},$$
$$\frac{d\hat{b}_{2_z}}{dz} = -i\kappa(z)\hat{b}_{2_z} + ig\hat{M}_{2_z}\hat{b}_{2_z} + \hat{f}_{1_z}, \quad (4)$$

where $\hat{M}_{1_z,2_z} = (1+\gamma)\hat{b}^\dagger_{1_z,1_z}\hat{b}_{1_z,1_z} + 2\hat{b}^\dagger_{2_z,2_z}\hat{b}_{2_z,2_z}$ and $\hat{N}_z = \hat{b}^\dagger_{1_z}\hat{b}_{1_z} + \hat{b}^\dagger_{2_z}\hat{b}_{2_z}$ =constant. The last terms $\hat{f}_{2_z} = ig(1-\gamma)\hat{b}^\dagger_{1_z}\hat{b}^2_{2_z}$ and $\hat{f}_{1_z} = ig(1-\gamma)\hat{b}^\dagger_{2_z}\hat{b}^2_{1_z}$ in the equations, represent external sources. If we consider that the variation of $\kappa(z)$ is much smaller than its average value, say κ_0 and that the input field amplitude is also small, the single-mode solution of equations (4) might be approximated by a solution of the second-order nonlinear coupled equations, in which the higher order source terms are neglected in comparison with the linear terms. This approximation has been used, in the sense of a rotating wave approximation, by several authors [33, 34] for slightly different problems. Originally, the bent coupler allows not very high variations from the average value (or central value associated with central mode [35]), otherwise, the exchange of the energy between two waveguides, because of overlapping of evanescent waves, does lose its meaning, i.e., we will not have the bent coupler anymore. In this way, we find similarly as in [20, 24] that

$$\hat{b}^\dagger_{1_z}\hat{b}_{1_z} \simeq \text{constant},$$
$$\hat{b}^\dagger_{2_z}\hat{b}_{2_z} \simeq \text{constant}, \quad (5)$$

thus, the solutions are easily found for a length L of the coupler in the form

$$\hat{b}_{1_L} = e^{iK(L)+ig\hat{M}_{1_z}L}\hat{b}_{1_0},$$
$$\hat{b}_{2_L} = e^{-iK(L)+ig\hat{M}_{2_z}L}\hat{b}_{2_0}; \quad (6)$$

where $K(L) = \int_0^L \kappa(L')dL'$ and z will be specified later. Returning to the original operators and taking $z = 0$, we will find an exact agreement with earlier results obtained in [20] where it was demonstrated that the codirectional geometry for the KNC exhibited various quantum properties, such as collapses and revivals of oscillations, squeezing of vacuum fluctuations and sub-Poissonian photon statistics in single as well as in compound modes. The field returns periodically to a pure state. The dynamics of energy exchange between waveguides for self trapping of beams were already discussed. We will not repeat all those calculations, instead we will pay more attention to the collapses and revivals of the field mode amplitudes, which are main quantum effects in the codirectional KNC [19]. Using the above operators the solution can be found in the form

$$\hat{a}_{1_L} = e^{i\hat{\theta}L}\left[(\cos\hat{\phi}(L))\hat{a}_1 + (\sin\hat{\phi}(L))\hat{a}_2\right],$$
$$\hat{a}_{2_L} = e^{i\hat{\theta}L}\left[(\cos\hat{\phi}(L))\hat{a}_2 + (\sin\hat{\phi}(L))\hat{a}_1\right], \quad (7)$$

where $\hat{\theta} = g(3+\gamma)\hat{N}_{z'}/2$ and $\hat{\phi}(L) = K(L) + gL(1-\gamma)(\hat{J}_{+_z} + \hat{J}_{-_z})/2$ and the angular momentum operators being $\hat{J}_{+_z} = \hat{a}^\dagger_{1_z}\hat{a}_{2_z} = \hat{J}_{-_z}$, $\hat{a}_{1_0,2_0} \equiv \hat{a}_{1,2}$. In the solution, we have assumed that the operators $\hat{a}_{1_z,2_z}$, $\hat{a}_{1_{z'},2_{z'}}$ and $\hat{a}_{1,2}$ are independently specified, i.e., they commute with each other. In this spirit, it is clear that operators (7) satisfy the commutation relations $[\hat{a}_{2_L,1_L}, \hat{a}^\dagger_{2_L,1_L}] = \hat{1}$ and the conservation law $\hat{a}^\dagger_{1_L}\hat{a}_{1_L} + \hat{a}^\dagger_2\hat{a}_2 = \hat{a}^\dagger_2\hat{a}_2 + \hat{a}^\dagger_1\hat{a}_1$. The number of these operators can be reduced due to the fact that the term $\hat{N}_{z'}$ is a constant, so that we can substitute the sub index by z in the expression of $\hat{\theta}$. However, we have found the quantum consistent solution for a coupler with the length L, using four independent operators $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_{1_z}$ and $\hat{a}_{2_z}$ $(z \neq 0, L)$, the expressions of operators $\hat{a}_{1_z}$ and $\hat{a}_{2_z}$ are not given yet. If nonlinear coupling coefficients g and $\bar{g}$ are very small comparing with linear coupling coefficient $\kappa(z)$, one can use linear approximations such that

$$\hat{a}_{1_z}|\alpha_1,\alpha_2\rangle \sim \hat{a}_1|\alpha_1,\alpha_2\rangle = \alpha_1|\alpha_1,\alpha_2\rangle,$$
$$\hat{a}_{2_z}|\alpha_1,\alpha_2\rangle \sim \hat{a}_2|\alpha_1,\alpha_2\rangle = \alpha_2|\alpha_1,\alpha_2\rangle, \quad (8)$$

to express phases $\hat{\theta}$ and $\hat{\phi}(L)$ in (7), when we need to calculate quantum statistical terms for the given input two mode coherent state $|\alpha_1,\alpha_2\rangle$. It is worth to note that using linear approximations (8), the solutions (7) are not necessarily written as the short-length approximation in which the exponential, cosine and sine functions are expanded into Taylor series in L, keeping only linear terms, i.e., the solutions (7) are found for arbitrary L. It is clearly seen from (5), after returning to the original operators that

$$\hat{J}_{+_z} + \hat{J}_{-_z} \simeq \hat{J}_{+_{z'}} + \hat{J}_{-_{z'}} \simeq \text{constant}, \quad (9)$$

using the fact

$$\hat{N}_z = \hat{N}_{z'} = \text{constant}. \quad (10)$$

Therefore (9) and (10) are valid even for a sufficiently short length z' $(z' \to 0)$ where $\hat{a}_{1_z,2_z} \sim \hat{a}_{1,2}$ as in (8), which are used in the solutions (7). Because of the operator form of the phase $\hat{\phi}(L)$ of the functions cosine and sine, it is natural to expect quantum effects, even, for the case that the one of eigenvalues of input operators is zero, when $\gamma \neq 1$. For instance, from the corresponding classical solution where we have to change the operators by c numbers, of course, the cosine and sine functions will not be affected by the input fields if one mode being in vacuum. This is a promising advantage of the presenting method. Let us investigate how the input vacuum in the second nonlinear waveguide is stimulated by the coherent state with an amplitude α_1 prepared in the first waveguide. The mean number of photons in single modes as a function of the length L is found to be

$$\begin{aligned} \bar{n}_1(L) &= \langle \hat{a}^\dagger_{1_L} \hat{a}_{1_L} \rangle = |\alpha_1|^2 \langle \cos^2 \hat{\phi}(L) \rangle, \\ \bar{n}_2(L) &= \langle \hat{a}^\dagger_{2_L} \hat{a}_{2_L} \rangle = |\alpha_1|^2 \langle \sin^2 \hat{\phi}(L) \rangle, \end{aligned} \quad (11)$$

fulfilling $\bar{n}_1(L)+\bar{n}_2(L) = |\alpha_1|^2$ =constant. Using the ordering theorem for the angular momentum operators [36], we obtain the expression (see Appendix)

$$\bar{n}_1(L) \simeq \frac{|\alpha_1|^2}{2}\left(1 + e^{-2\sin^2(gL(1-\gamma)/2)|\alpha_1|^2}\cos(2K(L))\right). \quad (12)$$

The assumption that one of the input modes being in vacuum allows us to conserve only terms (the exponential function in the front of the cosine in (12)) which appear from normal operator ordering procedures. This quantum term plays the key role in the collapses and revivals of output intensities dependent on the length of couplers.

As is seen from figure 4, the quantum evolution of the mean number of photons (12) is in a perfect agreement with that found by the standard method [20]. For bent couplers, as seen from (12) the collapses and revivals of the evolution of mean number of photons are developed in dependence on amplitudes and on nonlinear phases $2K(L)$, similarly as in figures demonstrated in [24]. In this way, the switching characteristics can be controlled by adjusting the form of coupling functions. The present method still can be employed for the contradirectional KNC, in order to extract, at least main quantum effects, in the sense of similarity [16] of codirectional and contradirectional KNC.

B. Contradirectional nonlinear couplers

The contradirectional KNC is composed of two parallel nonlinear waveguides with the third order nonlinear susceptibility, whose forward and backward propagating modes mutually exchange the energies by means of evanescent waves. The equations of motion of this Kerr coupler, in the interaction picture, are given by [26]

$$\begin{aligned} \frac{d\hat{a}_{1_z}}{dz} &= i\kappa(z)\hat{a}_{2_z} + 2ig(\hat{a}^\dagger_{1_z}\hat{a}_{1_z})\hat{a}_{1_z} + i\bar{g}(\hat{a}^\dagger_{2_z}\hat{a}_{2_z})\hat{a}_{1_z}, \\ \frac{d\hat{a}_{2_z}}{dz} &= -i\kappa(z)\hat{a}_{1_z} - 2ig(\hat{a}^\dagger_{2_z}\hat{a}_{2_z})\hat{a}_{2_z} - i\bar{g}(\hat{a}^\dagger_{1_z}\hat{a}_{1_z})\hat{a}_{2_z}, \end{aligned} \quad (13)$$

where $\hat{N}_z = \hat{a}^\dagger_{1_z}\hat{a}_{1_z} - \hat{a}^\dagger_{2_z}\hat{a}_{2_z}$ =constant and mode 2 is backpropagating. To obtain equations (13), we have used the standard rule [13] for quantum contradirectional couplers, changing the sign of the derivative for the backward beam in (13). Similarly as for the codirectional KNC, substituting new bosonic operators $\hat{b}_{1_z,2_z} = \frac{1}{\sqrt{2}}(\hat{a}_{1_z} \pm i\hat{a}_{2_z})$, (13) can be rewritten in the form

$$\begin{aligned} \frac{d\hat{b}_{1_z}}{dz} &= \kappa(z)\hat{b}_{1_z} + ig\hat{M}_{1_z}\hat{b}_{1_z} + \hat{f}_{2_z}, \\ \frac{d\hat{b}_{2_z}}{dz} &= -\kappa(z)\hat{b}_{2_z} + ig\hat{M}_{2_z}\hat{b}_{2_z} + \hat{f}_{1_z}, \end{aligned} \quad (14)$$

where $\hat{M}_{1_z,2_z} = (1-\gamma)\hat{b}^\dagger_{2_z,1_z}\hat{b}_{1_z,2_z} + 2\hat{b}^\dagger_{1_z,2_z}\hat{b}_{2_z,1_z}$ and $\hat{N}_z = \hat{b}^\dagger_{1_z}\hat{b}_{2_z} + \hat{b}^\dagger_{2_z}\hat{b}_{1_z}$ =constant. The last term $\hat{f}_{2_z} = ig(1+\gamma)\hat{b}^\dagger_{2_z}\hat{b}^2_{2_z}$ ($\hat{f}_{1_z} = ig(1+\gamma)\hat{b}^\dagger_{1_z}\hat{b}^2_{1_z}$) in the equation for the first (second) mode defines a third-order nonlinear coupling of the second (first) mode $\hat{b}_{2_z}$ ($\hat{b}_{1_z}$), representing an external source, again. Therefore, it can be assumed that the single-mode solution of equations (14) may be approximated by a solution of the second order nonlinear coupled equations, in which the higher-order source terms of the other mode are neglected, so that

$$\begin{aligned} \hat{b}^\dagger_{1_z}\hat{b}_{2_z} &\simeq \text{constant}, \\ \hat{b}^\dagger_{2_z}\hat{b}_{1_z} &\simeq \text{constant}. \end{aligned} \quad (15)$$

Thus the two equations become decoupled in terms of $\hat{b}_{1_z,2_z}$, but, of course, the equations for $\hat{a}_{1_z,2_z}$ are coupled. The decoupled solutions are given by

$$\begin{aligned} \hat{b}_{1_L} &= e^{K(L)+ig\hat{M}_{1_z}L}\hat{b}_{1_0}, \\ \hat{b}_{2_L} &= e^{-K(L)+ig\hat{M}_{2_z}L}\hat{b}_{2_0} \end{aligned} \quad (16)$$

for a given length L; here z has been chosen such that $z \neq 0, L$. For contradirectional couplers, we assume that the first mode is specified on the left-hand side of one nonlinear waveguide, $\hat{a}_{1_0} = \hat{a}_1$, and the second mode is specified on the right hand side of the other waveguide, $\hat{a}_{2_L} = \hat{a}_2$, thus having

$$\begin{aligned} \hat{a}_{1_L} &= \hat{C}^{-1}(\hat{C}^2 - \hat{C}\hat{S}\hat{C}^{-1}\hat{S})\hat{a}_1 + i\hat{S}\hat{C}^{-1}\hat{a}_2, \\ \hat{a}_{2_0} &= \hat{C}^{-1}\hat{a}_2 + i\hat{C}^{-1}\hat{S}\hat{a}_1, \end{aligned} \quad (17)$$

where $\hat{C} = \frac{1}{2}\left(e^{K(L)}e^{ig\hat{M}_{1_z}L} + e^{-K(L)}e^{ig\hat{M}_{2_z}L}\right)$ and $\hat{S} = \frac{1}{2}\left(e^{K(L)}e^{ig\hat{M}_{1_z}L} - e^{-K(L)}e^{ig\hat{M}_{2_z}L}\right)$ with

$\hat{M}_{1_z,2_z} = (3-\gamma)\hat{J}_{3_z} \mp i(1+\gamma)(\hat{J}_{+_z} + \hat{J}_{-_z})/2$, where the angular momentum operators $\hat{J}_{3_z} = \frac{1}{2}(\hat{a}^{\dagger}_{1_z}\hat{a}_{1_z} - \hat{a}^{\dagger}_{2_z}\hat{a}_{2_z})$, $\hat{J}_{+_z} = \hat{a}^{\dagger}_{1_z}\hat{a}_{2_z} = \hat{J}^{\dagger}_{-_z}$ satisfy the commutation relations $[\hat{J}_{3_z}, \hat{J}_{\pm_z}] = \pm\hat{J}_{\pm}$, and $[\hat{J}_{+_z}, \hat{J}_{-_z}] = 2\hat{J}_{3_z}$. Provided that for contradirectional couplers $\hat{N}_z = 2\hat{J}_{3_z}$ is the constant of motion for any z, we can write $\hat{J}_{3_z} \equiv \hat{J}_{3_{z'}}$, so $\hat{J}_{3_{z'}}$ commutes with all other operators with $z' \neq z$. Due to the fact that $[\hat{C}, \hat{S}] = \hat{0}$, solutions (17) can be rewritten as

$$\begin{aligned}\hat{a}_{1_L} &= e^{i\hat{\theta}L}\mathrm{sech}(\hat{\phi}(L))\hat{a}_1 + i\tanh(\hat{\phi}(L))\hat{a}_2,\\ \hat{a}_{2_0} &= e^{-i\hat{\theta}L}\mathrm{sech}(\hat{\phi}(L))\hat{a}_2 + i\tanh(\hat{\phi}(L))\hat{a}_1, \quad (18)\end{aligned}$$

where $\hat{\theta} = g(3-\gamma)\hat{J}_{3_{z'}} \equiv g(3-\gamma)\hat{J}_{3_z}$ and $\hat{\phi}(L) = K(L) + gL(1+\gamma)(\hat{J}_{+_z} + \hat{J}_{-_z})/2$. The operators (18) satisfy the commutation relations $[\hat{a}_{2_0,1_L}, \hat{a}^{\dagger}_{2_0,1_L}] = \hat{1}$ and the conservation law $\hat{a}^{\dagger}_{1_L}\hat{a}_{1_L} - \hat{a}^{\dagger}_2\hat{a}_2 = \hat{a}^{\dagger}_{2_0}\hat{a}_{2_0} - \hat{a}^{\dagger}_1\hat{a}_1$. Four independent operators $\hat{a}_{1,2}$, $\hat{a}_{1_z,2_z}$ are again used in the solutions (18) for an arbitrary L, where

$$\begin{aligned}\hat{a}_{1_z}|\alpha_1,\alpha_2\rangle &\sim \hat{a}_1|\alpha_1,\alpha_2\rangle = \alpha_1|\alpha_1,\alpha_2\rangle,\\ \hat{a}_{2_z}|\alpha_1,\alpha_2\rangle &\sim \hat{a}_2|\alpha_1,\alpha_2\rangle = \alpha_2|\alpha_1,\alpha_2\rangle, \quad (19)\end{aligned}$$

using (15) or (9) and (10). It is worth to note that all above calculations will be accomplished exactly and simply in the special case when a self- and cross-nonlinearities compensate each other, where $\gamma = -1$, which is analogue of the condition $\gamma = 1$ for codirectional couplers [20]. In this case the forces $\hat{f}_{1_z,2_z}$ are exactly missing and one can also consider not only small variations of $\kappa(z)$ and, in addition, operator solutions (18) can be rewritten in terms of c-number hyperbolic functions, giving rise to the switching of two modes. This shows that the method adopted here is reasonable and, in general, we can expect nonclassical effects because of the operator form of hyperbolic functions involved.

C. Quantum effects in switching

For the sake of simplicity, we investigate how the input vacuum in the second nonlinear waveguide is stimulated by the coherent state with an amplitude α_1 prepared in the first waveguide. The mean numbers of photons in single modes as a function of the length L are found to be

$$\begin{aligned}\bar{n}_1(L) &= \langle\hat{a}^{\dagger}_{1_L}\hat{a}_{1_L}\rangle = |\alpha_1|^2\langle\mathrm{sech}^2(\hat{\phi}(L))\rangle,\\ \bar{n}_2(L) &= \langle\hat{a}^{\dagger}_{2_0}\hat{a}_{2_0}\rangle = |\alpha_1|^2\langle\tanh^2(\hat{\phi}(L))\rangle, \quad (20)\end{aligned}$$

fulfilling $\bar{n}_1(L) + \bar{n}_2(L) = |\alpha_1|^2$ =constant. Expanding the function $f(x) = \mathrm{sech}^2 r = 4/(x + \frac{1}{x} + 2)$ with $x = e^{-2r}$ $(r > 0)$ into Taylor series, and then using the ordering theorem for the angular momentum operators [36] and the Baker-Hausdorff formula for the second mode, we obtain the expression (see Appendix)

$$\begin{aligned}\bar{n}_1(L) \simeq 4|\alpha_1|^2 \textstyle\sum_{n=0}^{\infty}(-1)^n(n+1)\exp(-|2K(L)\\ \times(n+1) - (\mathrm{sech}\eta)(1-\mathrm{sech}\eta)|\alpha_1|^2|), \quad (21)\end{aligned}$$

where $\eta = gL|1+\gamma|(n+1)$. The assumption that one of the input modes being in vacuum, allows us to remain only terms which appear from normal operator ordering procedures. In the same way, we can calculate two mode quadrature (using the quadrature operators $\hat{X}_1 = \hat{a}_{1_L} + \hat{a}_{2_0} + \hat{a}^{\dagger}_{1_L} + \hat{a}^{\dagger}_{2_0}$ and $\hat{X}_2 = (\hat{a}_{1_L} + \hat{a}_{2_0} - \hat{a}^{\dagger}_{1_L} - \hat{a}^{\dagger}_{2_0})/i$) and principal squeeze variances [11]

$$\begin{aligned}&\langle(\Delta\hat{X}_{1,2}(L))^2\rangle = 2[1 + \langle\Delta\hat{a}^{\dagger}_{1_L}\Delta\hat{a}_{1_L}\rangle + \langle\Delta\hat{a}^{\dagger}_{2_0}\Delta\hat{a}_{2_0}\rangle\\ &+2\mathrm{Re}\langle\Delta\hat{a}^{\dagger}_{1_L}\Delta\hat{a}_{2_0}\rangle \pm \mathrm{Re}\,(\langle(\Delta\hat{a}_{1_L})^2\rangle) + \langle(\Delta\hat{a}_{2_0})^2\rangle\\ &\mathrm{Re}\,(\langle(\Delta\hat{a}_{1_L})^2\rangle) + \langle(\Delta\hat{a}_{2_0})^2\rangle + 2\,(\langle\Delta\hat{a}_{1_L}\Delta\hat{a}_{2_0}\rangle)],\end{aligned} \quad (22)$$

$$\begin{aligned}\lambda(L) &= 2[1 + \langle\Delta\hat{a}^{\dagger}_{1_L}\Delta\hat{a}_{1_L}\rangle + \langle\Delta\hat{a}^{\dagger}_{2_0}\Delta\hat{a}_{2_0}\rangle\\ &+ 2\mathrm{Re}\langle\Delta\hat{a}^{\dagger}_{1_L}\Delta\hat{a}_{2_0}\rangle - |\langle(\Delta\hat{a}_{1_L})^2\rangle + \langle(\Delta\hat{a}_{2_0})^2\rangle\\ &+ 2\langle\Delta\hat{a}_{1_L}\Delta\hat{a}_{2_0}\rangle|]\end{aligned} \quad (23)$$

where, for two operators $\hat{a}$, $\hat{b}$, we denote $\langle\Delta\hat{a}\Delta\hat{b}\rangle = \langle\hat{a}\hat{b}\rangle - \langle\hat{a}\rangle\langle\hat{b}\rangle$. It is also worth to calculate the quadrature uncertainty product

$$u(L) = \langle(\Delta\hat{X}_1)^2\rangle\langle(\Delta\hat{X}_2)^2\rangle \geq 4,$$

testing generation of minimum uncertainty states. Fluctuations of the integrated intensity in single modes defined by the fourth-order moments in the field operators are given as [11]

$$\begin{aligned}\langle(\Delta W_{1_L,2_0})^2\rangle &= \langle\hat{a}^{\dagger 2}_{1_L,2_0}\hat{a}^2_{1_L,2_0}\rangle - \langle\hat{a}^{\dagger}_{1_L,2_0}\rangle^2\langle\hat{a}_{1_L,2_0}\rangle^2\\ &= |\alpha_1|^4(\langle\mathrm{sech}^4(\hat{\phi}(L))\rangle - \langle\mathrm{sech}^2(\hat{\phi}(L))\rangle^2) \quad (24)\end{aligned}$$

Correlations of integrated intensity fluctuations in different modes are obtained as [11]

$$\langle\Delta\left[\hat{a}^{\dagger}_{1_L}\hat{a}_{1_L}\right]\Delta\left[\hat{a}^{\dagger}_{2_0}\hat{a}_{2_0}\right]\rangle \equiv -\langle(\Delta W_{1_L,2_0})^2\rangle, \quad (25)$$

verifying that the total variance is

$$\langle(\Delta W_{1_L})^2\rangle + \langle(\Delta W_{2_0})^2\rangle + 2\langle\Delta\left[\hat{a}^{\dagger}_{1_L}\hat{a}_{1_L}\right]\Delta\left[\hat{a}^{\dagger}_{2_0}\hat{a}_{2_0}\right]\rangle = 0, \quad (26)$$

which indicates that the whole field is Poissonian (while sub Poissonian statistics in single modes can be observed). Thus the nonlinear interaction modifies only the phase of the compound field leading to squeezing of vacuum fluctuations, whereas the photon statistics are Poissonian, as is typical for the Kerr effect [37]. Let us consider two forms of linear couplings, similarly as in [25]: (1) the raised cosine function

$$\kappa(z) = \kappa_0 + \Delta \cos\left(\frac{2\pi m(z - L_{max})}{L_{max}}\right) \quad (27)$$

with m being an integer number, $\Delta < \kappa_0$ and L_{max} is the longest length of interest; (2) the raised Gaussian function

$$\kappa(z) = \kappa_0 + \Delta \exp\left(-\frac{\sigma(z - L_{max})^2}{2}\right) \quad (28)$$

with a variance $1/\sigma$ ($\sigma \neq 0$). In the case of the bent coupler the Gaussian variations of coupling coefficient (28) can be approximately obtained [23]. For the bent coupler with the length L_{max} we choose interaction length L (or coupler with the length L) to be centered, so that input output ports are situated at the distance $L/2$ to the left and right sides from the centre.

In figure ?? we have depicted the statistical quantities in dependence on the length L of conventional, raised cosine and bent couplers. For conventional couplers we see from the first column of figure ?? (also figures in [26]) that the switching of the modes is accompanied by an amplitude collapse around $L = 1$, when the initial zero fluctuations in single modes increase to their maximum, representing photon bunching, whereas, the correlation of these single mode fluctuations decreases to its minimum, showing the mode anticorrelation (see the second panel of the first column). At the same time, we see from the third panel of the first column that two mode squeezing is exhibited, since $\langle(\Delta \hat{X}_2)^2\rangle < 2$ for $L \simeq 1$. In the following stage, the correlation of single mode fluctuations increases, while quantum fluctuations in single modes reduce and the variances are negative (see the second panel of the first column) showing photon antibunching and finally, they approach again zero value corresponding to the coherent state.

It is also worth to plot extinction ratios of couplers as a function of L, where the extinction ratio is defined similarly as in [25]

$$\text{Ext.Ratio} = 10\left|\text{Log}_{10}\left(\frac{\bar{n}_2(L)}{\bar{n}_1(L)}\right)\right|.$$

In figure ?? we have plotted extinction ratios for $\gamma = -1$ (solid curves). In this classical case the zero extinction ratio is a point at which the outputs from both waveguides are equal. It is also called the critical point [25]. As is seen from figure ??, bent couplers provide faster switching, in contrast to raised cosine couplers giving slower switching than conventional couplers, because their critical points appear on the left and right handsides of the critical point of the conventional couplers. In addition, monotonical increase indicates that all the input energy in the first waveguide is transferred to the second waveguide, field of which is approaching switched pure states, since the arguments of the logarithm tends to the infinity. In the quantum case, in the region of collapses, the zero ratio defines curves dropped on the axes (see dotted curves of figure ??) so that the definition of the critical point fails as is mentioned in the classical case.

III. CONCLUSION

We have investigated quantum statistical properties of directional couplers composed of two Kerr nonlinear waveguides, in general, with variations of linear coupling coefficients. We have adopted Heisenberg equations of motion and the appropriate operator substitutions which enable us to obtain weakly coupled equations, whereas the higher order nonlinear coupling terms of the other modes in the exact equations are neglected in comparison with linear terms and their solutions are found. We have shown that the results obtained by means of the approximate method, coincide, perfectly, with those obtained by the standard method for the codirectional as well as bent KNC. In doing so, we argue that even for contradirectional couplers, this method does still work as well, in order to find, at least, the main quantum effects such as collapses and revivals of the field mode amplitudes. Using quantum consistent solutions, we have calculated the mean number of photons, fluctuations of the integrated intensity and their correlation in single modes and have demonstrated the quantum effect of collapse during switching of the modes, accompanied by enhanced squeezed quantum fluctuations. Two mode quadrature squeezing and sub-Poissonian photon statistics are also obtained. Finally, we have predicted the presumable external control of switching of the modes and the corresponding quantum effects, such as quadrature squeezing and photon antibunching by adjusting the forms for varying linear coupling coefficients.

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APPENDIX A: APPLICATIONS OF THE OPERATOR-ORDERING THEOREM

In accordance with calculations of mean number of photons, let us adopt the ordering theorem for the angular momentum operators

$$\exp(\lambda_+ \hat{J}_{+_z} + \lambda_- \hat{J}_{-_z}) = \exp(\Lambda_+ \hat{J}_{+_z})\exp[(\ln\Lambda_{3_z})\hat{J}_{3_z}] \cdot\exp(\Lambda_- \hat{J}_{-_z}) \equiv \exp(\Lambda_- \hat{J}_{-_z})\exp[(-\ln\Lambda_3)\hat{J}_{3_z}] \cdot\exp(\Lambda_+ \hat{J}_{+_z}), \tag{A1}$$

where

$$\Lambda_\pm = \frac{\lambda_\pm}{\eta}\tanh\eta,\ \Lambda_3 = (\cosh\eta)^{-2},\ \eta^2 = \lambda_+\lambda_-. \tag{A2}$$

In calculations of mean number of photons (11) for codirectional KNC, it is needed to find $\langle\cos^2(\hat{\phi}(L))\rangle$, where $\hat{\phi}(L) = K(L) + gL(1-\gamma)(\hat{J}_{+_z} + \hat{J}_{+_z})/2$. One reads that

$$\langle\cos^2(\hat{\phi}(L))\rangle = \frac{1}{2}(1 + \langle\cos(2\hat{\phi}(L))\rangle) = \frac{1}{4}(2 + \langle e^{2i\hat{\phi}(L)}\rangle + \langle e^{-2i\hat{\phi}(L)}\rangle), \tag{A3}$$

where the exponents can be expanded, by using the antinormal ordering formula from (A1) and using the fact that $\langle\alpha_1,0|\exp(\Lambda_-\hat{J}_{-_z}) \simeq \langle\alpha_1,0|$ and $\exp(\Lambda_+\hat{J}_{+_z})|\alpha_1,0\rangle \simeq |\alpha_1,0\rangle$ (remember that $\hat{a}_{1_z,2_z} \sim \hat{a}_{1,2}$), in the form

$$\langle e^{2i\hat{\phi}(L)}\rangle \simeq e^{2iK(L)}\langle e^{igL(1-\gamma)(\hat{J}_{+_z}+\hat{J}_{+_z})/2}\rangle = e^{2iK(L)}\langle e^{(-\ln\Lambda_3)\hat{J}_{3_z}}\rangle, \tag{A4}$$

where $\Lambda_3 = \cos^{-2}(gL(1-\gamma))$. Then remembering that $\hat{J}_{3_z} = \hat{a}^\dagger_{1_z}\hat{a}_{1_z} - \hat{a}^\dagger_{2_z}\hat{a}_{2_z}$ and again using normal ordering theorem for boson operators, we get

$$\langle e^{\pm 2i\hat{\phi}(L)}\rangle \simeq e^{\pm 2iK(L)}e^{-2(\sin^2 gL(1-\gamma)/2)|\alpha_1|^2}. \tag{A5}$$

Therefore (A3) is given by

$$\langle\cos^2(\hat{\phi}(L))\rangle \simeq \frac{1}{2}(1+e^{-2\sin^2(gL(1-\gamma)/2)|\alpha_1|^2}\cos(2K(L))). \tag{A6}$$

For contradirectional couplers, in the expression for mean number of photons (20) we have to calculate $\langle\mathrm{sech}^2(\hat{\phi}(L))\rangle$ with $\hat{\phi}(L) = K(L) + gL(1+\gamma)(\hat{J}_{+_z} + \hat{J}_{+_z})/2$. Expanding the function $f(x) = \mathrm{sech}^2 r = 4/(x + \frac{1}{x} + 2)$ with $x = e^{-2r}$ $(r > 0)$ into Taylor series as

$$\langle\mathrm{sech}^2(\hat{\phi}(L))\rangle = 4\sum_{n=0}^{\infty}(-1)^n(n+1)\langle e^{-2\hat{\phi}(L)(n+1)}\rangle, \tag{A7}$$

we have

$$\langle\mathrm{sech}^2(\hat{\phi}(L))\rangle = 4\sum_{n=0}^{\infty}(-1)^n(n+1)e^{-2K(L)(n+1)} \langle e^{\Lambda\hat{J}_{+_z}}e^{(\ln\Lambda_3)\hat{J}_{3_z}}e^{\Lambda\hat{J}_{-_z}}\rangle; \tag{A8}$$

here the coefficients read $\eta = gL|1+\gamma|(n+1)$, $|\Lambda| = \tanh\eta$, $\Lambda_3 = \mathrm{sech}^2\eta$. Note that in this case we have used normal ordering formula, otherwise the second term expressed as $\exp(-\ln\Lambda_3)\hat{J}_{3_z}$ diverges when $n \to \infty$. Due to the fact that $\hat{J}_{3_z}$ =constant, the second exponent in (A8) may be put on the left-hand side changing its sub index and returning again to z and it holds that

$$\langle\mathrm{sech}^2(\hat{\phi}(L))\rangle \simeq 4\sum_{n=0}^{\infty}(-1)^n(n+1)e^{-2K(L)(n+1)} e^{(\exp(\ln\sqrt{\Lambda_3})-1)|\alpha_1|^2}\langle e^{\Lambda\rho\hat{a}_2}e^{\Lambda\hat{a}_2^\dagger\alpha_1}\rangle. \tag{A9}$$

Using the Baker Hausdorff formula for the second mode, we obtain the expression (21).

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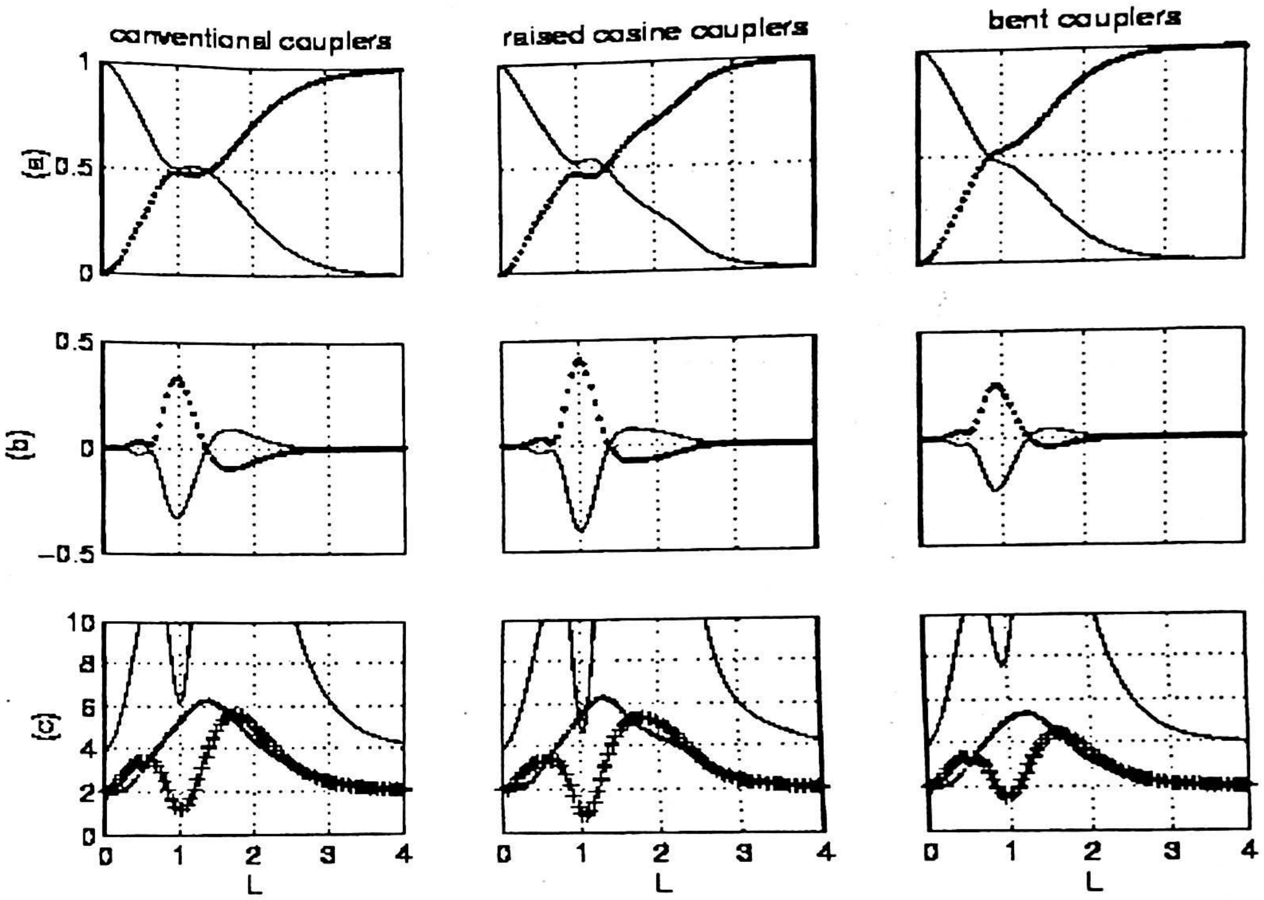

FIG. 1: Quantum statistical characteristics in contradirectional Kerr nonlinear couplers as functions of the length L with the nonlinear couplings $g = 0.24$, $\gamma = 1$ and linear coupling $\kappa(L) = \kappa_0 = 1$ (conventional couplers), $\kappa(L) = \kappa_0 + \Delta\cos(2\pi m(L - L_{max})/L_{max})$, $\Delta = 0.3$, $m = 3$, $L_{max} = 4$ (raised cosine couplers) and $\kappa(L) = \kappa_0 + \Delta\exp(-\sigma(L - L_{max})^2/2)$, $\sigma = 1$ (bent couplers) for input two-mode coherent state $|\alpha_1, \alpha_2\rangle$, $\alpha_1 = 2.5$, $\alpha_2 = 0$. (a) Evolutions of the normalized mean number of photons $\bar{n}_1(L)/|\alpha_1|^2$ (solid curves) and $\bar{n}_2(L)/|\alpha_1|^2$ (dotted curves); (b) the normalized fluctuations $\langle(\Delta W_{1_L,2_L})^2\rangle/|\alpha_1|^4$ (dotted curves) of integrated intensities and their correlations $\langle\Delta\left[\hat{a}^{\dagger}_{1_L}\hat{a}_{1_L}\right]\Delta\left[\hat{a}^{\dagger}_{2_0}\hat{a}_{2_0}\right]\rangle/|\alpha_1|^4$ (solid curves); (c) two-mode quadrature variances $\langle(\Delta\hat{X}_1)^2\rangle$ (dotted curves), $\langle(\Delta\hat{X}_2)^2\rangle$ (plus marked curves) and their uncertainty products $u(L)$ (solid curves).

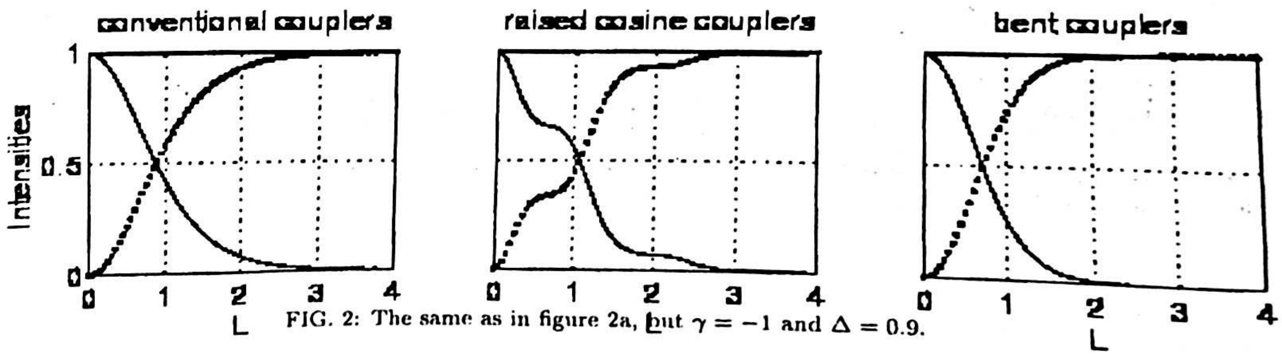

FIG. 2: The same as in figure 2a, but $\gamma = -1$ and $\Delta = 0.9$.

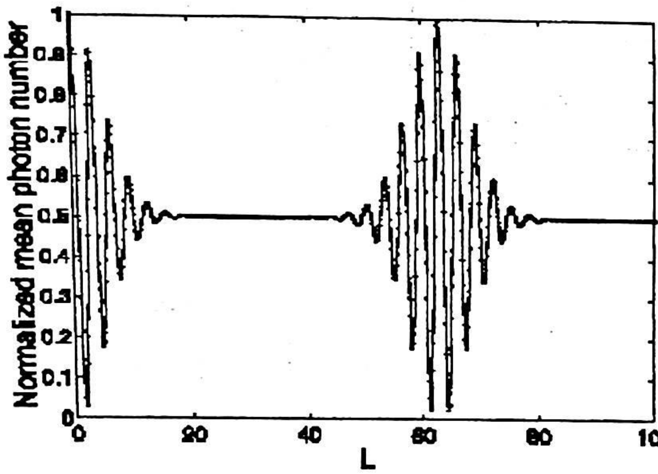

FIG. 3: A quantum evolution of the normalized mean number of photons in the first mode in conventional codirectional Kerr nonlinear couplers as a function of the length L with the nonlinear couplings $g = 0.1$, $\gamma = 0$ and linear coupling $\kappa(L) = \kappa_0 = 1$, for input two mode coherent state $|\alpha_1, \alpha_2\rangle$, $\alpha_1 = 2$, $\alpha_2 = 0$, using the standard (dotted curve) and the present (solid curve) methods.

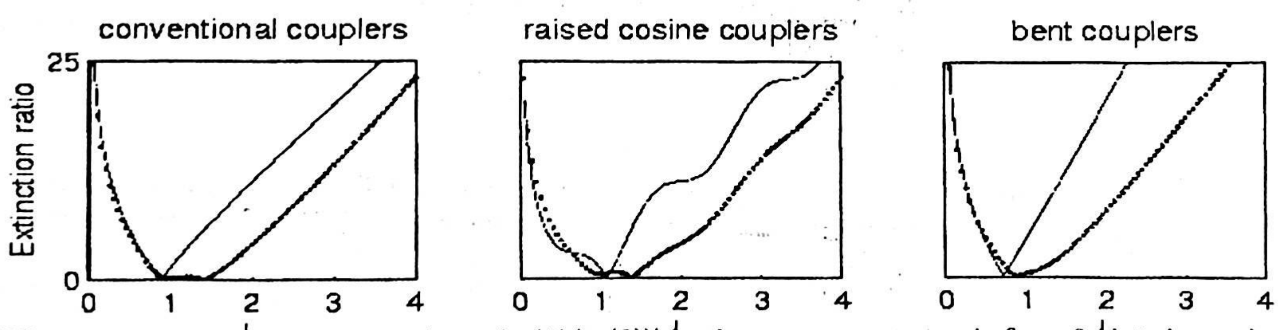

FIG. 4: Extinction ratios defined by $10|\mathrm{Log}_{10}(\bar{n}_2(L)/\bar{n}_1(L))|$ for the same parameters as in figure 2 (dotted curves) and in figure 3 (solid curves).