AN ESSAY ON THE MONGOLIAN MONETARY POLICY*

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Abstract

In this essay, we proposed two hypothesis on the Mongolian monetary policy rule. In order to answer the hypothesis we estimate a New Keynesian dynamic stochastic general equilibrium (DSGE) model of a small open economy (SOE) via the Bayesian estimation technique. We use the posterior odds test focusing on the modified generic Taylor-rule monetary policy, where the monetary authority reacts in response to inflation deviations from inflation target rates, output gaps, and exchange-rate movements. The main result is that the central bank of Mongolia (Bank of Mongolia - BoM) do not concern inflation target rates and systematically respond to nominal exchange rate (NER) changes when setting its monetary policy rule. We also find that terms-of-trade (ToT) movements do not contribute significantly to domestic business cycles.

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Абстракт

Энэ эссенд Монгол Улсын мөнгөний бодлогын дүрэмтэй холбоотой хоёр таамаглал дэвшүүлсэн. Хариулт өгөхийн тулд бид жижиг нээлттэй эдийн засгийн Шинэ Кейнсийн Динамик Стохастик Ерөнхий Тэнцвэрийн (ДСЕТ) загварыг үнэлнэ. Мөнгөний бодлого хэрэгжүүлэгчид инфляцийн зорилтот түвшнөөсөө хазайх хазайлт, гарцын зөрүү болон валютын ханшны хөдөлгөөнд хариу үзүүлдэг гэсэн засварлагдсан ерөнхий Тайлор-дүрмийн (Taylor-rule) мөнгөний бодлого дээр төвлөрсөн "posterior odds" тестийг ашиглана. Хамгийн гол үр дүн нь Монгол Улсын төв банк (Монгол Банк) нь мөнгөний бодлогын дүрмээ тогтоохдоо инфляцийн зорилтот түвшинг харгалзан үздэггүй/тооцдоггүй бөгөөд валютын нэрлэсэн ханшны өөрчлөлтөнд системтэйгээр хариу үйлдэл үзүүлдэг. Мөн, худалдааны нөхцлийн хөдөлгөөн нь дотоодын эдийн засгийн мөчлөгт статистикийн хувьд найдвартайгаар нөлөөлдөггүй.

1 Introduction

A recent trend in the monetary policy research is to use a generic Taylor-rule for the setting of interest rate policy. The Taylor-rule theory and the New Keynesian macroeconomic theory constitute the new macroeconomic research framework named as DSGE models. According to Clarida (2014), the Taylor-rule framework is a convergence result of many theories for conducting, evaluating monetary policy over the past twenty years. In DSGE models with nominal rigidities, flexible exchange rates and inflation targeting produce desirable macroeconomic results in open economies. Moreover, with its crucial advantages, the Taylor-rule framework will be dominating theory for monetary policy research. Following the influential work of Smets and Wouters (2003) and Adolfson et al. (2008), the central banks are building and estimating their DSGE models with nominal rigidities and are using them for monetary policy analysis.

As the DSGE models have the power to explain monetary policy implications and business cycles of a country, it would be important and interesting to apply this research framework to the monetary policy of the BoM. To do so, we reviewed the recent Mongolian monetary policy facts and obtained the following two issues that can motivate this research.

First, Mongolia has been pursuing a form of implicit or informal inflation targeting framework from 2000s. As mentioned in Hammond (2012) and other similar documents, recently there are 27 countries in the world have a formal inflation targeting regime. On other hand, in every end of year, the Parliament of Mongolia resolves the annual Monetary Guidelines which includes the next year's inflation target rate[1] and a provision that the BoM mandatorily to follow or concern this rate on their policy setting. From this conflicted fact, we can realize our first research issue that whether the BoM really concern this inflation target rates on its monetary policy rule setup or not. The result would be useful for future monetary policy settings and to find optimal policy rule for the current economic situations.

Second, the recent official exchange rate regime - a managed floating by the BoM and a floating by the IMF - is actually effective in the Mongolian economy? Calvo and Reinhart (2002) show that most exchange rate

[1]In the online appendix, Table A1 summarizes inflation target rates, monetary policy and exchange rate regimes of Mongolia over the observation period, 2000-2014.

regimes described as a floating under the IMF classification, are actually characterized by heavy exchange rate management by the monetary policy authorities. Using exchange rates, NIRs, international reserves and commodity prices as indicators of policy intervention and external shocks, they demonstrate that a floating regimes of most emerging market economies more closely utilize a fixed exchange rate regimes than actual float. We have calculated the probabilities of variability of the interest rates and the international reserves of Mongolia[2] by following the approach in the article. As our results, Mongolia has much more variability than in the US which is considered as a pure floating regime. It means that Mongolia may not a floating regime and may be a PEG as a shown in the graph since it is located more close to the PEG.

On the other hand, the exchange rate is one of the important ingredients of monetary policy when a country chooses from the non-fixed exchange rate regimes. As discussed in Taylor (2001), the long-run monetary policy in a such country is based on the trinity of (i) a flexible exchange rate, (ii) an inflation target, and (iii) a monetary policy rule. These policy implications differ to each other based on the issue about how exchange rates should be include in monetary policy and how should the instruments of monetary policy (the interest rate or a monetary aggregate) react to the exchange rate. According to Lubik and Schorfheide (2007), these issues can be transferred to an important research question of what extent a central bank responds to exchange rate movements when making monetary policy? Answers to these two questions may be one because a pure floating means that a central bank do not respond systematically to exchange rate movements and vice versa.

Finally, we can summarize the main purpose of the essay is to answer these two questions by estimating a DSGE model of a SOE for Mongolia. For the theoretical framework, we use Lubik and Schorfheide (2007) which is derived from Gali and Monacelli (2005) that extend the benchmark New Keynesian DSGE model to a SOE setting. Open economies have a possibility to participate in inter-temporal as well as intra-temporal trade in order to keep consumption above and beyond what is possible in a closed economy. Moreover, foreign shocks, such as the terms of trade, can change domestic business cycle fluctuations which may lead the monetary authority

[2] The corresponding figures are in the online appendix.

to explicitly take into account international variables. The model consists of a forward-looking (open economy) dynamic IS equation (DIS) and a New Keynesian Phillips curve (NKPC) relationship. The DIS is derived from a consumption Euler equation when households consume both domestically produced and imported goods. The NKPC is obtained from the optimal price setting decisions of domestic producers. Monetary policy is described by the modified Taylor-type rule, while the exchange rate is introduced via the definition of the consumer price index (CPI) and under the assumption of purchasing power parity (PPP).

The essay is organized as follows. The section 2 summarizes the related literature review. In section 3, the structural SOE model is derived from the mentioned DSGE model, which we proceed to estimate. In Section 4, we discuss the estimation approach - Bayesian method, estimation results, and the results on the proposed hypothesis testings. Section 5 contains our conclusions.

2 Literature review

In this essay, we use the following three broad concepts, i) DSGE modeling, ii) Bayesian estimation and inference, and iii) some related empirical facts of Mongolia; therefore, it may more convenient to organize this section by these three parts.

2.1 DSGE modeling

As described in Negro and Schorfheide (2010) the DSGE models are a research framework to study macroeconomic issues in dynamic horizon. It implies that the main decision rules of economic agents are originated from the solution of inter-temporal optimization problems as same as in the RBC theory. In economy, there are also many uncertainties, for example total factor productivity, nominal interest rates and its deviations, that can influence agents, and these uncertainties are usually generated from exogenous stochastic processes.

According to Gali (2008), the New Keynesian (NK) and the Real Business Cycle (RBC) theories are the most influential developments in macroeconomics for the last three decades. The RBC revolution had a impact on both of methodological and conceptual areas, and the most important one is that the RBC theory constituted the use of dynamic stochastic general equilibrium (DSGE) models as "workhorse" for macroeconomic analysis. However, in the empirical area or among the central banks and other policy institutions, the RBC approach and its version with money referred as to the classical monetary model were not perceived as yielding a framework that was relevant for policy analysis. This kind of model generally predicts neutrality (or near neutrality) of monetary policy with respect to real variables, such as output and employment. That result is an opposite to central bankers' view that monetary policy has an influence output and employment, at least in the short run. Moreover, the classical monetary models generally yield a normative implication that the only one optimal monetary rule is to keep the short term nominal rate constant at a zero level (the Friedman rule) even though this policy is not consistent with the implementing desirable monetary policy by the central banks. The conflict between theoretical predictions and evidence, and between normative implications and policy practice, can be viewed as a symptom that some elements that are important in actual economies may be missing in classical monetary models. Those shortcomings are the main motivation behind the introduction of some Keynesian assumptions, while maintaining the RBC apparatus as an underlying structure.

As concluded in many recent research studies, the New Keynesian framework is established to understand relationship between monetary policy, inflation, and the business cycle and has been the main tool for the recent research on the theory and practice of monetary policy. Recently, this framework has been used to research on monetary policy in the open economy as well.

2.2 Bayesian estimation technique and inference

As mentioned in Herbst and Schorfheide (2016), the Bayesian technique has been used as an estimation tool for DSGE models since 15 years ago and examples of pioneers are DeJong et al. (2000), Schorfheide (2000), and Otrok (2001). To date, DSGE models cover a broad area of macroeconomic research fields in particular monetary policy issues, and consequently the literature is becoming an abundant.

Geweke et al. (2011) summarizes the main important contributions of Bayesian analysis and explains a rapid growth of estimated DSGE models as follows.

First, Smets and Wouters (2003) is the one of influential research works that shows how to derive a DSGE model from the neoclassical growth model. It improves the model by introducing a habit formation in consumption, capital adjustment costs, variable factor utilization, nominal price and wage stickiness, behavioral rules for government spending and monetary policy. DSGE models are usually criticized on their fitting and forecasting performance of key macroeconomic variables, but by introducing potential exogenous shocks into the model, these disadvantages could be solved that is comparable to VAR and make DSGE models a powerful competitor within macroeconomic research frameworks. Bayesian methods updates estimation results using non-sample information, which is through specification of prior distributions, is one reason to use it widely.

Second, the many latest researches have devoted to invent the importance of various pass-through mechanisms that are useful for explaining empirical facts of business cycle fluctuations. The posterior odd test procedure that is based on Bayesian posterior model probabilities are commonly used to compare competing model specifications. One of a good example is Rabanal and Rubio-Ramirez (2005) which shows how to use this comparison method for determining the relative importance of wage and price rigidities. We can use it for a comparison analysis even if the model specifications are non-nested, for example, a DSGE model with sticky wages versus a DSGE model with sticky prices.

Finally, DSGE models with nominal rigidities are becoming a "workhorse" for a monetary policy research. According to Adolfson et al. (2007), many central banks of the world have been using DSGE models as their main research framework. This kind of models usually have a unique stable rational expectations solution for the main monetary policy rule coefficients that are satisfying the following common properties: i) to maximize the welfare of a representative consumer or minimize a inflation and output gap, ii) to determine welfare maximizing mechanism between the state variables of the economy and the monetary policy instruments. The key elements for the determination of such optimal policy problems are always unknown parameters of firm's technology and consumer's preference. Then, the main advantage of the Bayesian method is determined as availability for researchers to find these parameters through maximizing expected posterior welfare.

2.3 Empirical facts of the Mongolian monetary policy

As a result of the democratic revolution and transition to the market economy in 1990, a two-tiered banking system, which is comprising of the (central) BoM and commercial banks, established in 1991. The main objective of the BoM's monetary policy is to sustain stability of national currency Togrog in the external and internal markets. The stability of Togrog refers to the stable exchange rate in the external market and to the stable CPI or price stability in the domestic market.

As published in the official website of the BoM[3], the BoM had a monetary aggregate targeting framework in between mid of 1990s and mid 2000s. In this period, the BoM was implementing policy by controlling reserve money as the operating target and M2 as the intermediate target. However, since mid 2000s, the BoM have faced the difficulties on implementing this type of policy due to the instability on the velocity of money, money demand, and money multiplier resulting from the ongoing remonetization process in the economy. Because of these difficulties, the BoM has been shifting their monetary policy to inflation targeting framework since 2007 based on the mid-term plan. By the this plan, the intermediate target of the framework is inflation rates and the final purpose is the stability of price.

In order to achieve desired objective, the BoM has been trying to implement the following conceptions under the inflation targeting monetary policy framework: i) announcing mid-term targeted inflation to the public, ii) defining price stability as the BoM's main and long-term objective of monetary policy and taking every possible measures to maintain inflation rate within its targeted range, iii) utilizing all available information (not only regarding monetary aggregates) in the process of monetary policy decision-making, iv) ensuring transparency of the monetary policy strategy by publicizing and introducing the objectives and operational plans of the monetary policy-makers, and v) coordinating the responsibility of the BoM with inflation performance.

The announcement of mid-term inflation target rate to the public is one of the main components of the this policy framework. The main purpose are i) to reduce agent's uncertainty for the decision making process and

[3]https://mongolbank.mn/eng/listmonetarypolicy.aspx

ii) to ensure the central bank's transparency tend to be have conventional channels to deliver their decision plans to the public.

Moreover, the BoM has used one-week central banks' bill as the policy rate since 2007. By managing the policy rate, the BoM can influence on the expectations of the deposit rate and thus the lending rate of commercial banks. Policy rate movements are an indicator of the monetary policy direction (easing or tightening) and thus it is also main leading factor for interbank market rates. It means that the weighted average rate of interbank market trading is expected to be an approximately same level of the policy rate. Based on this correlation, when the economy have a high inflationary pressure the BoM increase policy rate intends to slow down the rapid growth of monetary and credit aggregates, to keep them at an optimal level and to avoid overheating. In contrast, when the economy faces difficulties on the economic growth, the BoM lower policy rate or the cost of money in order to support loans and spending, to recover the economy.

3 A small open economy model

In this section we show how to derive the linear DSGE model in Lubik and Schorfheide (2007) from the small open economy model in Gali and Monacelli (2005). We estimate this model by using Mongolian data in the next section. We use model explanations in Gali and Monacelli (2005) without any changes, but we add some derivations of equations basing in Bergholt (2012).

The model has four sectors of households, firms, monetary authority and foreign economy. It assumes that the world economy consists of a continuum of small open economies represented by the unit interval. Every single economy has zero share of world economy, so its domestic policy decisions do not have any impact on the rest of the world. However, different economies are correlated through productivity shocks, and they have identical preferences, technology, and market structure. Notice that goods produced in home country denoted with subscript h, imported goods related variables denoted with subscript f, and foreign economy variables are denoted with superscript*.

3.1 Households

The domestic economy is inhabited by a representative household who attempts to maximize her lifetime utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where N_t is labor hours and, C_t is a consumption bundle; σ is the inverse elasticity of inter-temporal substitution and φ is the inverse elasticity of labour supply to real wage.

The consumption bundle, C_t is defined as a composite consumption index defined by a constant elasticity of substitution (CES) form,

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where η is the elasticity of substitution of domestic goods to foreign goods, from the side of the domestic consumer; $\alpha \in [0,1]$ is share of imported consumption goods and inverse related to the degree of home bias in preferences, and is thus a natural index of openness. $C_{h,t}$ and $C_{f,t}$ are indices of domestic goods and foreign goods, which both are given by the CES functions,

$$C_{h,t} \equiv \left(\int_0^1 C_{h,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{f,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where $j \in [0,1]$ denotes the good variety. $C_{i,t}$ is, in turn, an index of the quantity of goods imported from country i and consumed by domestic households. It is given by an analogous CES function:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where parameter $\varepsilon > 1$ denotes the elasticity of substitution between varieties (produced within any given country).

Utility maximization problem of (1) subjects to a sequence of budget constraints of the form

$$\int_0^1 P_{h,t}(j)C_{h,t}(j)dj+\int_0^1\int_0^1 P_{i,t}(j)C_{i,t}(j)djdi+E_t\left\{Q_{t,t+1}D_{t+1}\right\}\leq D_t+W_tN_t+T_t \tag{3}$$

for all t. The domestic price on good j denoted $P_{h,t}(j)$ while the price on good j imported from country i is denoted $P_{i,t}(j)$. D_{t+1} is the nominal payoff in period $t+1$ from a portfolio held at the end of period t. $Q_{t,t+1}$ is the stochastic discount factor for one-period forward nominal payoffs of the domestic household. The nominal wage is denoted W_t while lump-sum transfers/taxes is denoted T_t. In here, domestic currency is a common measurement of these variables.

We assume that households can access completely to international financial markets and have a complete set of contingent claims. It implies that monetary policy can be specified in terms of an interest rate rule directly and indirectly. Thus, we do not need to introduce money explicitly in either the utility function or budget constraint.

In order to use the budget constraint to maximization problem, first we need to determine the demand functions based on the optimal allocation of any given expenditure within each category as follows[4]. The optimal demand for home good j:

$$C_{h,t}(j)=\left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}C_{h,t} \tag{4}$$

In a similar way, the aggregate price index for imported goods from country i is given by:

$$P_{i,t}\equiv\left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$$

The optimal consumption of good j imported from country i is given by:

$$C_{i,t}(j)=\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon}C_{i,t} \tag{5}$$

[4]The details are in the online appendix.

The aggregate price index for all imported goods is given by:

$$P_{f,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

The optimal basket of import consumption from country i is:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{f,t}} \right)^{-\gamma} C_{f,t} \tag{6}$$

Finally, the aggregate consumption price index (CPI) in the home country is given by:

$$P_t \equiv \left[(1-\alpha) P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

It follows from (4) - (6) that[5]

$$\int_0^1 P_{h,t}(j) C_{h,t}(j) dj = P_{h,t} C_{h,t}; \qquad \int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t}$$

and

$$\int_0^1 P_{i,t} C_{i,t} di = P_{f,t} C_{f,t}.$$

By analogously, the optimal allocation of expenditures between domestic and imported goods is determined by

$$C_{h,t} = (1-\alpha) \left(\frac{P_{h,t}}{P_t} \right)^{-\eta} C_t; \qquad C_{f,t} = \alpha \left(\frac{P_{f,t}}{P_t} \right)^{-\eta} C_t \tag{7}$$

Notice that, when the price indexes for domestic and foreign goods are equal (as in the steady state described below), parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness.

It follows from (7) and the given CPI definition that

$$P_{h,t} C_{h,t} = (1-\alpha) P_{h,t}^{1-\eta} P_t^{\eta} C_t; \qquad P_{f,t} C_{f,t} = \alpha P_{f,t}^{1-\eta} P_t^{\eta} C_t$$

[5]The details are in the online appendix.

$$\begin{aligned} P_{h,t}C_{h,t} + P_{f,t}C_{f,t} &= (1-\alpha)P_{h,t}^{1-\eta}P_t^{\eta}C_t + \alpha P_{f,t}^{1-\eta}P_t^{\eta}C_t \\ &= P_t^{\eta}C_t \underbrace{\left[(1-\alpha)P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta}\right]}_{=P_t^{1-\eta}} \\ &= P_t^{\eta}C_tP_t^{1-\eta} = P_tC_t \end{aligned}$$

If we combine above results into the period budget constraint definition we have

$$\begin{aligned} D_t + W_tN_t + T_t &\geq \underbrace{\int_0^1 P_{h,t}(j)C_{h,t}(j)dj}_{=P_{h,t}C_{h,t}} + \int_0^1 di \underbrace{\int_0^1 P_{i,t}(j)C_{i,t}(j)dj}_{=P_{i,t}C_{i,t}} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \\ &\geq P_{h,t}C_{h,t} + \underbrace{\int_0^1 P_{i,t}C_{i,t}di}_{=P_{f,t}C_{f,t}} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \\ &\geq P_{h,t}C_{h,t} + P_{f,t}C_{f,t} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \\ &\geq P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \end{aligned}$$

Then, the aggregated household maximization problem becomes

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

such that

$$P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t \tag{8}$$

We can get the following optimilaity conditions and a conventional stochastic Euler equation by using the standart solution approach to this optimization problem[6]:

$$C_t^{\sigma}N_t^{\varphi} = \frac{W_t}{P_t} \tag{9}$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{10}$$

$$\beta R_t E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\} = 1 \tag{11}$$

[6]The details are in the online appendix.

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross yield on a risk-less one-period bond paying off one unit of domestic currency in $t+1$ (with $E_t\{Q_{t,t+1}\}$ being its price).

Then (9) and (11) can be respectively written in log-linear form as[7]:

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \rho\right) \qquad (12)
\end{aligned}
$$

where lower case letters denote the logs of the respective variables, $\rho = -\ln\beta$, which is the usual definition of the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \ln P_t$). The nominal interest rate (NIR) is defined here as $r_t = \ln(R_t) = -\ln(E_t\{Q_{t,t+1}\})$.

3.2 The inflation, the exchange rate, and the terms of trade

3.2.1 The terms of trade

Bilateral terms of trade between the domestic economy and country i is defined as the price of country i's goods in terms of home goods:

$$\mathcal{S}_{i,t} = \frac{P_{i,t}}{P_{h,t}}$$

The effective terms of trade are thus given by:

$$\mathcal{S}_t \equiv \frac{P_{f,t}}{P_{h,t}} = \frac{\left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}}{P_{h,t}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_{h,t}}\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \qquad (13)$$

A first-order approximation around a symmetric steady state satisfying $\mathcal{S}_{i,t} = \mathcal{S}_i = 1$ for $\forall i$ gives us[8]:

$$\Rightarrow s_t = p_{f,t} - p_{h,t} \approx \ln\left(\int_0^1 \mathcal{S}_{i,t} di\right) \approx \int_0^1 s_{i,t} di \qquad (14)$$

[7]The details are in the online appendix.
[8]The details are in the online appendix.

where $s_t \equiv p_{f,t} - p_{h,t}$ denotes the log-linear effective terms of trade, i.e. the price of foreign goods in terms of home goods.

3.2.2 Domestic and CPI inflation

If we describe the log-linear form of the CPI around the same symmetric steady state satisfying the PPP condition $P_{h,t} = P_{f,t} = P$, we have[9]

$$P_t \equiv \left[(1-\alpha)P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

$$\begin{aligned} \Rightarrow p_t &\equiv (1-\alpha)p_{h,t} + \alpha p_{f,t} \\ &= p_{h,t} + \alpha s_t \end{aligned} \tag{15}$$

Domestic inflation is defined as the rate of change in the index of domestic goods prices:

$$\pi_{h,t} \equiv p_{h,t} - p_{h,t-1}$$

Thus, using (15) CPI inflation is given by:

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t \tag{16}$$

It shows that the difference between domestic inflation and CPI inflation is proportional to the percentage change in ToT and the index of openness α (the coefficient of proportionality).

3.2.3 The nominal and real exchange rate (RER)

Define $\mathcal{E}_{i,t}$ as the bilateral NER, i.e. the price of country i's currency in terms of domestic currency and $P_{i,t}^i(j)$ is the price of country i's good j expressed in the producer's (i.e. country i's) currency. Thus, $\mathcal{E}_{i,t}$ measures how many domestic currency units one country i's currency unit is worth. Assume that the law of one price holds for individual goods at all times for both import and export prices. Thus, for all goods $j \in [0,1]$ in every country $i \in [0,1]$:

$$P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^i(j)$$

[9] The details are in the online appendix.

where $P_{i,t}^{i} \equiv \left(\int_0^1 P_{i,t}^{i}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ is defined as the aggregate price level in country i in terms of country i currency, i.e. country i's domestic price index.

Aggregation across all goods using a price index for goods imported from country i: $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ gives:

$$\begin{aligned}
P_{i,t} &= \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}} \\
&= \left[\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^{i}(j)\right)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} = \left[\mathcal{E}_{i,t}^{1-\varepsilon}\int_0^1 P_{i,t}^{i}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \\
&= \mathcal{E}_{i,t}\left(\int_0^1 P_{i,t}^{i}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}} \\
&= \mathcal{E}_{i,t}P_{i,t}^{i}
\end{aligned}$$

In turn, by substituting into the definition of $P_{f,t}$ and transforming in log-linear form around the symmetric steady state, $\mathcal{E}$ and P^{i}, we obtain[10]

$$P_{f,t} = \left(\int_0^1 P_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^{i}\right)^{1-\gamma}di\right)^{\frac{1}{1-\gamma}}$$

$$\begin{aligned}
p_{f,t} &\approx \int_0^1 \left(e_{i,t}+p_{i,t}^{i}\right)di = \int_0^1 e_{i,t}di + \int_0^1 p_{i,t}^{i}di \\
&\approx e_t + p_t^{*}
\end{aligned}$$

where $e_t \equiv \int_0^1 e_{i,t}di$ is the (log) nominal effective exchange rate, $p_{i,t}^{i} \equiv \int_0^1 p_{i,t}^{i}(j)dj$ is the (log) domestic price index for country i (expressed in terms of its currency), and $p_t^{*} \equiv \int_0^1 p_{i,t}^{i}di$ is the (log) world price index. Notice that for the world as a whole there is no distinction between CPI and domestic price level, nor for their corresponding inflation rates.

Combining the previous result with the definition of the terms of trade we obtain the relationship between home and world price:

$$s_t = e_t + p_t^{*} - p_{h,t} \tag{17}$$

[10]The details are in the online appendix.

Next, we derive a relationship between the ToT and the RER. Define the bilateral RER with country i as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$, i.e. the ratio of the two countries' CPIs, both expressed in domestic currency. Let $q_t \equiv \int_0^1 q_{i,t} di$ be the (log) effective RER, where $q_{i,t} \equiv \ln \mathcal{Q}_{i,t}$. It follows that

$$\begin{aligned} q_t &= \int_0^1 \left(e_{i,t} + p_t^i - p_t \right) di \\ &= e_t + p_t^* - p_t \\ &= s_t + p_{h,t} - p_t \\ &= (1 - \alpha) s_t \end{aligned} \tag{18}$$

where the last equality holds only up to a first order approximation when $\eta \neq 1$.

3.3 International financial market

3.3.1 International risk sharing

Under the assumption of complete securities markets for securities traded internationally, a condition analogous to (10) must also hold for the representative household in any other country, say country i:

$$1 = \beta E_t \left\{ Q_{t,t+1}^{-1} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) \right\} \tag{19}$$

Divide (10) by (19) and solve for C_t[11]:

$$C_t = \vartheta^i C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{20}$$

for all t. $\vartheta_i = E_t \left\{ \frac{C_{t+1}}{C_{t+1}^i (\mathcal{Q}_{t+1}^i)^{\frac{1}{\sigma}}} \right\}$ is some constant which will generally depend on initial conditions regarding relative net asset positions. For simplicity and generality, we assume that there is symmetric initial conditions, for example zero net foreign assets and same expected conditions. This implies $\vartheta_i = \vartheta = 1$ for all i. Then, if we take logs on both sides of (20) we have:

$$c_t = c_t^i + \frac{1}{\sigma} q_{i,t} \tag{21}$$

[11]The details are in the online appendix.

Equation (21) is determined at the household level. Note that world consumption is given by $c^* \equiv \int_0^1 c_t^i di$. Integrating (21) over all i and using $q_t \equiv \int_0^1 q_{i,t} di$ and (18) yields:

$$\begin{aligned} c_t &= \int_0^1 \left(c_t^i + \frac{1}{\sigma} q_{i,t} \right) di = c_t^* + \frac{1}{\sigma} q_t \\ &= c_t^* + \left(\frac{1-\alpha}{\sigma} \right) s_t \end{aligned} \tag{22}$$

This equation express the relationship between domestic and world consumption by the ToT under an assumption of complete markets at the international level. It shows that if the ToT increases which means that domestic price to world price decreases, domestic consumption would be increased.

3.3.2 Uncovered interest parity (UIP) and the ToT

Allow households to invest both in domestic and foreign bonds; B_t and B_t^*. The budget constraint may be written as:

$$P_t C_t + Q_{t,t+1} B_{t+1} + Q_{t,t+1}^* \mathcal{E}_{t+1} B_{t+1}^* \leq B_t + \mathcal{E}_t B_t^* + W_t N_t + T_t$$

The optimality conditions with respect to these assets are:

$$1 = \beta E_t \left\{ Q_{t,t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{23}$$

$$1 = \beta E_t \left\{ \left(Q_{t,t+1}^* \right)^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right\} \tag{24}$$

Divide (23) by (24) to obtain[12]:

$$\frac{R_t}{R_t^*} = E_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}, \text{where } R_t = \frac{1}{E_t \left\{ Q_{t,t+1} \right\}} \tag{25}$$

Transforming to the log-linear form of (25) gives:

$$r_t - r_t^* = E_t \left\{ e_{t+1} - e_t \right\} = E_t \left\{ \Delta e_{t+1} \right\} \tag{26}$$

[12]The details are in the online appendix.

Now, from (17) we have that:

$$
\begin{aligned}
E_t s_{t+1} - s_t &= E_t e_{t+1} - e_t + E_t p^*_{t+1} - p^*_t - E_t p_{h,t+1} + p_{h,t} \\
&= E_t \left\{\Delta e_{t+1}\right\} + E_t \left\{\Delta \pi^*_{t+1}\right\} - E_t \left\{\pi_{h,t+1}\right\}
\end{aligned}
$$

$$
\Rightarrow s_t = -E_t \left\{\Delta e_{t+1}\right\} - E_t \left\{\Delta \pi^*_{t+1}\right\} + E_t \left\{\pi_{h,t+1}\right\} + E_t \left\{s_{t+1}\right\}
$$

Thus, using (26) we get the following stochastic difference equation:

$$
s_t = \left(r^*_t - E_t \left\{\pi^*_{t+1}\right\}\right) - \left(r_t - E_t \left\{\pi_{h,t+1}\right\}\right) + E_t \left\{s_{t+1}\right\} \tag{27}
$$

Given that the terms of trade are pinned down uniquely in the perfect foresight steady state, and given the assumptions of stationary in the models driving forces and unit relative prices in steady state, it follows that $\lim_{T\to\infty} E_t \left\{s_T\right\} = 0$. Hence, (27) can be solved forward to obtain:

$$
\begin{aligned}
s_t = {} & \left(r^*_t - E_t \left\{\pi^*_{t+1}\right\}\right) - \left(r_t - E_t \left\{\pi_{h,t+1}\right\}\right) + E_t \left\{s_{t+1}\right\} + \\
& + E_t \left\{\left(r^*_{t+1} - E_t \left\{\pi^*_{t+2}\right\}\right) - \left(r_{t+1} - E_t \left\{\pi_{h,t+2}\right\}\right) + \left(r^*_{t+2} - E_t \left\{\pi^*_{t+3}\right\}\right) - \right. \\
& \left. - \left(r_{t+2} - E_t \left\{\pi_{h,t+3}\right\}\right) + \ldots\right\}
\end{aligned}
$$

$$
\Rightarrow s_t = E_t \left\{\sum_{k=0}^{\infty} \left[\left(r^*_{t+k} - \pi^*_{t+k+1}\right) - \left(r_{t+k} - \pi_{h,t+k+1}\right)\right]\right\} \tag{28}
$$

Equation (28) expresses the terms of trade as the expected sum of real interest rate (RIR) differentials between the world market and the home market.

3.4 Firms

3.4.1 Technology

A domestic firm produces a differentiated good with a linear technology represented by the production function

$$
Y_t(j) = A_t N_t(j) \tag{29}
$$

where $j \in [0,1]$ is a firm-specific index and $a_t \equiv \ln A_t$ follows the $AR(1)$ process $a_t = \rho_1 a_{t-1} + \varepsilon_{a,t}$. The real marginal cost (expressed in terms of domestic prices) will be common across domestic firms and defined by[13]:

$$mc_t = -\nu + w_t - p_{h,t} - a_t \tag{30}$$

where $\nu \equiv \ln(1-\tau)$, with $\tau = \frac{1}{\varepsilon}$ being an employment subsidy.

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent an index for aggregate domestic output, analogous to the one introduced for consumption. If we assume that the market clearing in the labor market, we have

$$N_t \equiv \int_0^1 N_t(j)dj$$

In order to find the approximate aggregate production function we will rearrange the production function as follows:

$$Y_t(j) = A_t N_t(j) \quad \Rightarrow \quad N_t(j) = \frac{Y_t(j)}{A_t}$$

So,

$$\begin{aligned} N_t = \int_0^1 N_t(j)dj &= \int_0^1 \frac{Y_t(j)}{A_t} dj = \frac{1}{A_t}\int_0^1 Y_t(j)dj \\ &= \frac{Y_t}{A_t}\int_0^1 \frac{Y_t(j)}{Y_t} dj = \frac{Y_t Z_t}{A_t} \end{aligned}$$

where $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj$. Thus,

$$Y_t = \frac{A_t N_t}{Z_t}$$

and the log-linear form becomes:

$$y_t = a_t + n_t - z_t$$

where $z_t = \ln \int_0^1 \frac{Y_t(j)}{Y_t} dj$.

In the Appendix 2, we showed that $z_t \approx 0$ because $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj \approx 1$ up to a first-order approximation around $P_{h,t}(j) = P_{h,t}$. Thus, the above log-linear aggregate production function becomes:

[13] The details are in the online appendix.

$$y_t = a_t + n_t \tag{31}$$

3.4.2 Price-setting

Following staggered price setup in Calvo (1983), define θ the probability for a firm of keeping the price fixed and $(1-\theta)$ the probability for a firm of changing the price. In other words, in each period there is a constant probability $(1-\theta)$ that the firm will be able to adjust its price, independently of past history. Since we assume a continuum of firms of measure one, by the law of large numbers it follows that the fraction of retailers setting their price at t is $(1-\theta)$. Thus, only a fraction of firms is setting its price at a certain period in time allowing for inflation dynamics.

We use Appendix B of Gali and Monacelli (2005) and home firm's optimal price is determined by the following rule:

$$\bar{p}_{h,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ mc_{t+k}^n \right\} \tag{32}$$

for all t. $\bar{p}_{h,t}$ denotes the (log) of newly set domestic prices, and $\mu \equiv \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the log of the steady state mark-up.

We can see from (32) that firms will set price that corresponds to the desired mark-up plus a weighted average of their current and expected nominal marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon θ^k.

3.5 Market equilibrium

3.5.1 Demand side: Aggregate demand and output

Market clearing for good j in the home economy implies:

$$Y_t(j) = C_{h,t}(j) + \int_0^1 C_{h,t}^i(j)di \tag{33}$$

The supply of domestically produced good j is denoted $Y_t(j)$, the domestic demand is denoted $C_{h,t}(j)$, and country i's demand for good j produced in the home economy is denoted $C_{h,t}^i(j)$ for all $j \in [0,1]$ and all t. Due to the nested structure one can express demand in sub-markets in terms of total demand by combining all demand functions from each level. For instance, insert (7) into (4) and get:

$$C_{h,t}(j) = \left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon} C_{h,t} = (1-\alpha)\left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t \tag{34}$$

Furthermore, the demand for domestically produced good j in country i is expressed by nesting up across different demand layers in country i. First, note that the consumption of domestically produced good j in country i is a function of country i's consumption of goods produced in the home economy, given as in (4):

$$C^i_{h,t}(j) = \left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon} C^i_{h,t}$$

Second, note that country i's consumption of goods produced in the home economy is a function of country i's consumption of foreign goods, given as in (6):

$$C^i_{h,t} = \left(\frac{P_{h,t}}{\mathcal{E}_{i,t} P^i_{f,t}}\right)^{-\gamma} C^i_{f,t}$$

Third, note that consumption of imported goods in country i is a function of total consumption in that country, given as in (7):

$$C^i_{f,t} = \alpha\left(\frac{P^i_{f,t}}{P^i_t}\right)^{-\eta} C^i_t$$

Combining all these yields the demand for domestically produced good j in country i as a function of total consumption in that country:

$$C^i_{h,t}(j) = \alpha\left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}\left(\frac{P_{h,t}}{\mathcal{E}_{i,t} P^i_{f,t}}\right)^{-\gamma}\left(\frac{P^i_{f,t}}{P^i_t}\right)^{-\eta} C^i_t \tag{35}$$

Thus, we can insert (34) and (35) into (33) and get another form of domestic supply of goods j:

$$Y_t(j) = (1-\alpha)\left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t + \int_0^1 \alpha\left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}\left(\frac{P_{h,t}}{\mathcal{E}_{i,t}P^i_{f,t}}\right)^{-\gamma}\left(\frac{P^i_{f,t}}{P^i_t}\right)^{-\eta} C^i_t di$$
$$= \left(\frac{P_{h,t}(j)}{P_{h,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{h,t}}{\mathcal{E}_{i,t}P^i_{f,t}}\right)^{-\gamma}\left(\frac{P^i_{f,t}}{P^i_t}\right)^{-\eta} C^i_t di\right] \tag{36}$$

Plugging (36) into the definition of aggregate domestic output $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, we obtain[14]

$$Y_t = (1-\alpha)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{h,t}}{\mathcal{E}_{i,t}P^i_{f,t}}\right)^{-\gamma}\left(\frac{P^i_{f,t}}{P^i_t}\right)^{-\eta} C^i_t di$$
$$= \left(\frac{P_{h,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1 \left(\frac{\mathcal{E}_{i,t}P^i_{f,t}}{P_{h,t}}\right)^{\gamma-\eta} \mathcal{Q}^{\eta}_{i,t} C^i_t di\right]$$
$$= \left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1 \left(\mathcal{S}^i_t \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}^{\eta-\frac{1}{\sigma}}_{i,t} di\right] \tag{37}$$

where the last equality follows from (20), and where $(\mathcal{S}^i_t)$ denotes the effective terms of trade of country i, while $\mathcal{S}_{i,t}$ denotes the bilateral terms of trade between the home economy and foreign country i. Notice that in the particular case of $\sigma = \eta = \gamma = 1$ the previous condition can be written exactly as

$$Y_t = C_t \mathcal{S}^{\alpha}_t \tag{38}$$

More generally, and recalling that $\int_0^1 s^i_t di = 0$, we can derive the following first order log-linear approximation to (37) around the symmetric steady state:

$$y_t = c_t + \alpha\gamma s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t$$
$$= c_t + \frac{\alpha\omega}{\sigma} s_t \tag{39}$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Notice that $\sigma = \eta = \gamma = 1$ implies $\omega = 1$.

A condition analogous to the one above will hold for all countries. Thus, for a generic country i it can be rewritten as $y^i_t = c^i_t + \frac{\alpha\omega}{\sigma} s^i_t$. By aggregating

[14] The details are in the online appendix.

over all countries we can derive a world market clearing condition as follows:

$$\begin{aligned} y_t^* &\equiv \int_0^1 y_t^i di \\ &= \int_0^1 c_t^i di + \frac{\alpha\omega}{\sigma} \int_0^1 s_t^i di \\ &= \int_0^1 c_t^i di = c_t^* \end{aligned} \tag{40}$$

where $y_t^*$and c_t^* are indexes for world output and consumption (in log terms), and where the main equality follows, once again, from the fact that $\int_0^1 s_t^i di = 0$.

Combining (39) with (21) and (40), we obtain

$$\begin{aligned} y_t &= c_t^* + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha\omega}{\sigma} s_t = y_t^* + \frac{1-\alpha+\alpha\omega}{\sigma} s_t \\ &= y_t^* + \frac{(1-\alpha)+\alpha\omega}{\sigma} s_t \\ &= y_t^* + \frac{1}{\sigma_\alpha} s_t \end{aligned} \tag{41}$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.

Finally, combining (39) with Euler equation (12), we get

$$y_t - \frac{\alpha\omega}{\sigma} s_t = E_t \left\{ y_{t+1} - \frac{\alpha\omega}{\sigma} s_{t+1} \right\} - \frac{1}{\sigma} \left(r_t - E_t \left\{ \pi_{t+1} \right\} - \rho \right)$$

$$y_t = E_t \left\{ y_{t+1} \right\} - \frac{1}{\sigma} \left(r_t - E_t \left\{ \pi_{t+1} \right\} - \rho \right) - \frac{\alpha\omega}{\sigma} E_t \left\{ \Delta s_{t+1} \right\} \tag{42}$$

This IS equation is similar to the one in a closed economy except that now there is an additional term linking domestic output to the international environment. An alternative representation including domestic goods inflation is found by inserting (15) into (42):

$$\begin{aligned} y_t &= E_t \left\{ y_{t+1} \right\} - \frac{1}{\sigma} \left(r_t - E_t \left\{ \pi_{h,t+1} + \alpha \Delta s_{t+1} \right\} - \rho \right) - \frac{\alpha\omega}{\sigma} E_t \left\{ \Delta s_{t+1} \right\} \\ &= E_t \left\{ y_{t+1} \right\} - \frac{1}{\sigma} \left(r_t - E_t \left\{ \pi_{h,t+1} \right\} - \rho \right) - \frac{\alpha(\omega-1)}{\sigma} E_t \left\{ \Delta s_{t+1} \right\} \\ &= E_t \left\{ y_{t+1} \right\} - \frac{1}{\sigma} \left(r_t - E_t \left\{ \pi_{h,t+1} \right\} - \rho \right) - \frac{\alpha\Theta}{\sigma} E_t \left\{ \Delta s_{t+1} \right\} \end{aligned} \tag{43}$$

where $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$.

Inserting s_t from (41) into (43) we get [15]:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{h,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma}\sigma_\alpha E_t\left\{\left(y_{t+1} - y^*_{t+1}\right) - (y_t - y^*_t)\right\}$$
$$= E_t\{y_{t+1}\} - \frac{(r_t - E_t\{\pi_{h,t+1}\} - \rho)}{(\sigma - \alpha\Theta\sigma_\alpha)} + \frac{\alpha\Theta\sigma_\alpha E_t\left\{\Delta y^*_{t+1}\right\}}{(\sigma - \alpha\Theta\sigma_\alpha)}$$

Use $\Theta = \omega - 1$ and $\sigma_\alpha = \frac{\sigma}{1+\alpha(\omega-1)}$:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{h,t+1}\} - \rho) + \alpha\Theta E_t\left\{\Delta y^*_{t+1}\right\} \tag{44}$$

The expectation of the world output growth in one period forward, $E_t\{\Delta y^*_{t+1}\}$, is exogenous to domestic allocations. In general, the degree of openness α influences the responsibility of output to any given change in the domestic real rate $r_t - E_t\{\pi_{h,t+1}\}$. Also note from (43) that if $\Theta \equiv \omega - 1 > 0$ (i.e. if γ and η are sufficiently high) we have that $\sigma_\alpha = \frac{\sigma}{1+\alpha(\omega-1)} < \sigma$, and output is more responsible to real rate changes than in the closed economy case.

3.5.2 The trade balance

Next, we can define net exports nx_t as the difference between total domestic production and total domestic consumption, relative to steady state output Y:

$$nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{h,t}}C_t\right) \tag{45}$$

A first-order approximation around a symmetric steady state with price level $P_t = P_{h,t} = P$ and output level $Y_t = C_t = Y$, i.e. zero net export, yields:

$$nx_t \approx \frac{1}{Y}\left(Y - \frac{P}{P}Y\right) + \frac{1}{Y}\left[(Y_t - Y) - \frac{P}{P}(C_t - C) - \frac{1}{P}C(P_t - P) + \frac{1}{P^2}PC(P_{h,t} - P)\right]$$
$$= \frac{Y_t - Y}{Y} - \frac{C_t - C}{Y} - \frac{P_t - P}{P} + \frac{P_{h,t} - P}{P}$$
$$= (y_t - \not{y}) - (c_t - \not{c}) - (p_t - \not{p}) + (p_{h,t} - \not{p}) = y_t - c_t - p_t + p_{h,t}$$
$$= y_t - c_t - \alpha s_t (\text{using } (15))$$

[15] The details are in the online appendix.

which combined with (39) implies:

$$\begin{cases} nx_t &= y_t - c_t - \alpha s_t \\ y_t &= c_t + \dfrac{\alpha\omega}{\sigma} s_t \end{cases} \Rightarrow nx_t = \frac{\alpha\omega}{\sigma} s_t - \alpha s_t$$

$$\Rightarrow nx_t = \alpha\left(\frac{\omega}{\sigma} - 1\right) s_t \tag{46}$$

Again, in the special case of $\sigma = \eta = \gamma = 1$ we have $nx_t = 0$ for all t, though the later property will also hold for any configuration of those parameters satisfying $\sigma(\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = 0$. More generally, the sign of the relationship between the terms of trade and net exports is ambiguous, depending on the relative size of σ, γ, and η.

3.5.3 The supply side: Marginal cost and inflation dynamics

From the Appendix B of Gali and Monacelli (2005), we can see that the dynamics of domestic inflation in terms of real marginal cost are given as follows:

$$\pi_{h,t} = \beta E_t \{\pi_{h,t+1}\} + \lambda \widehat{mc}_t \tag{47}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$.

The real marginal cost is determined in our model as follows:

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{h,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{h,t}) - a_t \\ &= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\ &= -\nu + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t \end{aligned} \tag{48}$$

where (31) and (22) are used in the derivation.

An economy becomes an open implies that world prices and output have begun to influence home variables. As we can see from the equation, the ToT (home price relative to world price) and world output will increase home real marginal cost. Moreover, these two foreign variables influence on the home consumption and consequently, home labor supply will changed and so will the real wage. Technology and home output have an similar influences as in the closed economy, technology has a direct impact on labor productivity while home output level determines employment and the real wage.

Finally, using (41) to substitute for s_t, we can rearrange the previous expression in terms of the domestic output, world output, and technology:

$$mc_t = -\nu + (\sigma_\alpha + \varphi)y_t + (\sigma - \sigma_\alpha)y_t^* - (1+\varphi)a_t \tag{49}$$

Notice that in the special cases $\alpha = 0$ and/or $\sigma = \eta = \gamma = 1$, which imply $\sigma = \sigma_\alpha$, the domestic real marginal cost is completely insulated from movements in foreign output.

3.5.4 Equilibrium dynamics: the NKPC and the DIS

In this section we show that the linearized equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation dynamics. That representation, which we refer to as the canonical one, has provided the basis for the analysis and evaluation of alternative policy rules.

First, we define the domestic output gap x_t as the deviation of (log) domestic output y_t from its natural level y_t^n, where the latter is in turn defined as the equilibrium level of output in the absence of nominal rigidities (and conditional on world output y_t^*). Formally,

$$x_t \equiv y_t - y_t^n \tag{50}$$

The domestic natural level of output can be found after imposing $mc_t = -\mu$ for all t and solving for domestic output in equation (49):

$$-\mu = -\nu + (\sigma_\alpha + \varphi)y_t^n + (\sigma - \sigma_\alpha)y_t^* - (1+\varphi)a_t \tag{51}$$

Solve this for y_t^n and use that $\sigma_\alpha = \frac{\sigma}{1+\alpha\Theta}$:

$$(\sigma_\alpha + \varphi)y_t^n = \nu - \mu + (1+\varphi)a_t - (\sigma - \sigma_\alpha)\, y_t^*$$

$$y_t^n = \frac{\nu - \mu}{\sigma_\alpha + \varphi} + \frac{1+\varphi}{\sigma_\alpha + \varphi}a_t - \frac{\sigma - \dfrac{\sigma}{1+\alpha\Theta}}{(\sigma_\alpha + \varphi)}y_t^*$$

$$
\begin{aligned}
y_t^n &= \frac{\nu - \mu}{\sigma_\alpha + \varphi} + \frac{1+\varphi}{\sigma_\alpha + \varphi} a_t - \alpha \frac{\Theta \sigma_\alpha}{(\sigma_\alpha + \varphi)} y_t^* \\
\Rightarrow y_t^n &= \mathit{\Omega} + \Gamma a_t + \alpha \Psi y_t^* \qquad (52)
\end{aligned}
$$

where $\mathit{\Omega} \equiv \frac{\nu-\mu}{\sigma_\alpha+\varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Psi \equiv -\frac{\Theta\sigma_\alpha}{\sigma_\alpha+\varphi}$.

Second, if we subtract (51) from (49) gets the real marginal cost gap as follows:

$$
\begin{aligned}
\widehat{mc}_t &= -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1+\varphi) a_t - \\
&\quad - [-\nu + (\sigma_\alpha + \varphi) y_t^n + (\sigma - \sigma_\alpha) y_t^* - (1+\varphi) a_t] \\
&= (\sigma_\alpha + \varphi)(y_t - y_t^n)
\end{aligned}
$$

$$
\Rightarrow \widehat{mc}_t = (\sigma_\alpha + \varphi) x_t
$$

which we can combine with (47) to derive a version of the NKPC for the small open economy in terms of the output gap:

$$
\begin{aligned}
\pi_{h,t} &= \beta E_t\{\pi_{h,t+1}\} + \lambda(\sigma_\alpha + \varphi) x_t \\
&= \beta E_t\{\pi_{h,t+1}\} + \kappa_\alpha x_t \qquad (53)
\end{aligned}
$$

where $\kappa_\alpha \equiv \lambda(\sigma_\alpha + \varphi)$. Note that (53) nests the special case of a closed economy because $\alpha = 0$ implies that $\sigma_\alpha = \sigma$ (or $\sigma = \eta = \gamma = 1$) and then the slope coefficient is given by $\lambda(\sigma + \varphi)$ as in the standard, closed economy NKPC. In general, the relation between the degree of openness parameter α, an increase in the output gap, and domestic inflation, depends on the sign on Θ because $\sigma_\alpha = \frac{\sigma}{1+\alpha\Theta}$. If $\Theta > 0$ (i.e. if η and γ are sufficiently high), an increase in the openness will make domestic inflation less responsive to change in the output gap. On the other hand, if $\Theta < 0$, then more openness will make domestic inflation more responsive to output gap changes.

To derive the open economy DIS we define the RIR as

$$
rr_t = r_t - E_t \pi_{h,t+1}
$$

Then, IS equation given in (44) can be written as:

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left(r_t - E_t\{\pi_{h,t+1}\} - \rho\right) + \alpha\Theta E_t\{\Delta y^*_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(rr_t - \rho) + \alpha\Theta E_t\{\Delta y^*_{t+1}\} \end{aligned}$$

In similar way, the natural output is defined as a function of the natural RIR as follows:

$$y^n_t = E_t\{y^n_{t+1}\} - \frac{1}{\sigma_\alpha}(rr^n_t - \rho) + \alpha\Theta E_t\{\Delta y^*_{t+1}\} \tag{54}$$

The DIS yields by subtracting (54) from (44):

$$\begin{aligned} x_t = y_t - y^n_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left(r_t - E_t\{\pi_{h,t+1}\} - \rho\right) + \alpha\Theta E_t\{\Delta y^*_{t+1}\} - \\ - \left[E_t\{y^n_{t+1}\} - \frac{1}{\sigma_\alpha}(rr^n_t - \rho) + \alpha\Theta E_t\{\Delta y^*_{t+1}\}\right] \end{aligned}$$

$$\Rightarrow x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{h,t+1}\} - rr^n_t) \tag{55}$$

If we solve rr^n_t from (55) we have:

$$\begin{aligned} rr^n_t &= r_t - E_t\{\pi_{h,t+1}\} - \sigma_\alpha\left(E_t\{x_{t+1}\} - x_t\right) \\ &= r_t - E_t\{\pi_{h,t+1}\} - \sigma_\alpha\left(E_t\{y_{t+1} - y^n_{t+1}\} - (y_t - y^n_t)\right) \\ &= r_t - E_t\{\pi_{h,t+1}\} - \sigma_\alpha\left(E_t\{y_{t+1} - y_t\} - E_t\{y^n_{t+1} - y^n_t\}\right) \\ &= r_t - E_t\{\pi_{h,t+1}\} - \sigma_\alpha\left(E_t\{\Delta y_{t+1}\} - E_t\{\Delta y^n_{t+1}\}\right) \end{aligned}$$

From equation (52) and (44) we have $E_t\{\Delta y^n_{t+1}\} = \Gamma E_t\{\Delta a_{t+1}\} + \alpha\Psi E_t\{\Delta y^*_{t+1}\}$ and $E_t\{\Delta y_{t+1}\} = \dfrac{1}{\sigma_\alpha}\left(r_t - E_t\{\pi_{h,t+1}\} - \rho\right) - \alpha\Theta E_t\{\Delta y^*_{t+1}\}$, respectively, and by substituting these we get:

$$
\begin{aligned}
rr_t^n &= r_t - E_t\{\pi_{h,t+1}\} - \sigma_\alpha \bigg(\frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{h,t+1}\} - \rho) - \alpha\Theta E_t\{\Delta y^*_{t+1}\} - \\
&\qquad\qquad -\Gamma E_t\{\Delta a_{t+1}\} - \alpha\Psi E_t\{\Delta y^*_{t+1}\}\bigg) \\
&= \rho + \sigma_\alpha \Gamma E_t\{\Delta a_{t+1}\} + \alpha\sigma_\alpha(\Theta + \Psi)E_t\{\Delta y^*_{t+1}\} \\
&= \rho + \sigma_\alpha \Gamma E_t\{\rho_a a_t - a_t\} + \alpha\sigma_\alpha(\Theta + \Psi)E_t\{\Delta y^*_{t+1}\}
\end{aligned}
$$

$$
\Rightarrow rr_t^n = \rho - \sigma_\alpha\Gamma(1-\rho_a)a_t + \alpha\sigma_\alpha(\Theta + \Psi)E_t\{\Delta y^*_{t+1}\} \tag{56}
$$

Thus, we see that the NKPC and the DIS equations in the small open economy equilibrium is similar to the counterparts in the closed economy. A couple of differences appear however. First, the degree of openness influences the sensitivity of the output gap to interest rate changes. Second, openness generally makes the natural RIR depend on expected world output growth, in addition to domestic productivity.

3.6 A small, structural open economy model

In this section we summarize the above small, open model into the structural model, which is same in Lubik and Schorfheide (2007). The model consists of a forward-looking DIS equation and a NKPC. Monetary policy is described by the modified interest rate rule satisfied the current Mongolian monetary policy specifications. All exogenous shocks are assumed as given by the corresponding $AR(1)$ process, respectively. Moreover, we determine steady states of the model.

The DIS curve: Combining equation (16) into IS equation (44) we have:

$$
\begin{cases}
y_t = E_t\{y_{t+1}\} - \dfrac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{h,t+1}\} - \rho) + \alpha\Theta E_t\left\{\Delta y^*_{t+1}\right\} \\
\pi_t = \pi_{h,t} + \alpha\Delta s_t
\end{cases}
\Rightarrow
$$

$$
\begin{aligned}
\Rightarrow y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{t+1} - \alpha\Delta s_{t+1}\} - \rho) + \alpha\Theta E_t\left\{\Delta y^*_{t+1}\right\} \\
&= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha}{\sigma_\alpha}E_t\{\Delta s_{t+1}\} + \alpha\Theta E_t\left\{\Delta y^*_{t+1}\right\}
\end{aligned}
$$

where $\sigma_\alpha = \frac{\sigma}{1+\alpha\Theta}$ and $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1)$. If we denote as $\tau \equiv \frac{1}{\sigma}$ the inter-temporal substitution elasticity and assume that $\eta = \gamma = 1$ for simplicity we have:

$$\begin{aligned}
\alpha\Theta &= \alpha\left(\sigma - 1 + (1-\alpha)(\sigma-1)\right) = \alpha\left((\sigma-1)(1+1-\alpha)\right) \\
&= \alpha(\sigma-1)(2-\alpha) = \alpha(2-\alpha)\left(\frac{1}{\tau} - 1\right) \\
&= \alpha(2-\alpha)\left(\frac{1-\tau}{\tau}\right)
\end{aligned}$$

Then,

$$\begin{aligned}
\sigma_\alpha &= \frac{\sigma}{1+\alpha\Theta} = \frac{\sigma}{1+\alpha(\sigma-1)(2-\alpha)} = \frac{1}{\frac{1}{\sigma} + \alpha\frac{(\sigma-1)}{\sigma}(2-\alpha)} \\
&= \frac{1}{\tau + \alpha(1-\tau)(2-\alpha)}
\end{aligned}$$

As result of these calculations, the IS equation becomes:

$$\begin{aligned}
y_t = E_t\{y_{t+1}\} &- \left[\tau + \alpha(1-\tau)(2-\alpha)\right]\left(r_t - E_t\{\pi_{t+1}\} - \rho\right) - \\
&- \alpha\left[\tau + \alpha(1-\tau)(2-\alpha)\right] E_t\{\Delta s_{t+1}\} + \alpha(2-\alpha)\left(\frac{1-\tau}{\tau}\right) E_t\left\{\Delta y^*_{t+1}\right\}
\end{aligned} \tag{57}$$

As discussed in Lubik and Schorfheide (2005), in order to guarantee stationary of the model, real variables are expressed in terms of percentage deviations from A_t. Thus, we need to modify the IS becomes:

$$\begin{aligned}
y_t = E_t\{y_{t+1}\} &- \left[\tau + \alpha(1-\tau)(2-\alpha)\right]\left(r_t - E_t\{\pi_{t+1}\}\right) - \rho_a a_t - \\
&- \alpha\left[\tau + \alpha(1-\tau)(2-\alpha)\right] E_t\{\Delta s_{t+1}\} + \alpha(2-\alpha)\left(\frac{1-\tau}{\tau}\right) E_t\left\{\Delta y^*_{t+1}\right\}
\end{aligned} \tag{58}$$

The NKPC: If we combine (16) into the NKPC (53) we have:

$$\begin{aligned} \pi_{h,t} &= \beta E_t\{\pi_{h,t+1}\} + \lambda(\sigma_\alpha + \varphi)x_t \\ \pi_t - \alpha\Delta s_t &= \beta E_t\{\pi_{t+1} - \alpha\Delta s_{t+1}\} + \lambda\left(\frac{1}{\tau + \alpha(1-\tau)(2-\alpha)} + \varphi\right)x_t \\ \pi_t &= \beta E_t\{\pi_{t+1}\} - \alpha\beta E_t\{\Delta s_{t+1}\} + \alpha\Delta s_t + \frac{\kappa}{\tau + \alpha(1-\tau)(2-\alpha)}(y_t - y_t^n) \end{aligned} \tag{59}$$

where $y_t^n = -\alpha(2-\alpha)\left(\frac{1-\tau}{\tau}\right)y_t^*$ is potential output in the absence of nominal rigidities. The slope coefficient $\kappa > 0$ is a function of underlying structural parameters, such as labor supply and demand elasticities and parameters capturing the degree of price stickiness. Since we do not use any additional information from the underlying model we treat κ as structural.

Monetary policy rule: In order to complete or close the model, we need to determine the NIR. In here, we do not use the monetary rule function in Gali and Monacelli (2005) and Lubik and Schorfheide (2007) due to the our interesting hypothesis. We assume that the BoM follows a generalized Taylor-rule as in Smets and Wouters (2003) for deciding on the the interest rate. Based on the hypothesis and the Monetary Policy Guidelines of Mongolia, it is assumed that, in addition to smoothing the interest rate, $\rho_R r_{t-1}$, the interest rate is decided in reaction to CPI deviation from the inflation target, $\pi_{t-1} - \pi_t^T$, the output growth, Δy_t, and the nominal exchange rate changes, Δe_t. We also assume that there are two monetary policy shocks: one is a persistent shock to inflation target, which is assumed to follow a $AR(1)$ process $\pi_t^T = \rho_\pi \pi_{t-1}^T + \varepsilon_{\pi,t}$; the other is a temporary identically independent distributed (i.i.d) normal interest rate shock, $\varepsilon_{R,t}$. The latter will also be denoted a monetary policy shock. Then, the log-linear policy function for the BoM is given by

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left[\pi_t^T + \psi_1\left(\pi_{t-1} - \pi_t^T\right) + \psi_2\Delta y_t + \psi_3\Delta e_t\right] + \epsilon_{R,t} \tag{60}$$

In this specification, ψ_1, ψ_2 and ψ_3 are, respectively, the responses of the BoM to deviations of inflation from its target rates and the output growth, and smoothing nominal effective exchange rate volatility. As $\psi_1 \to \infty$ the central bank would be strictly targeting the inflation; or $\psi_2 \to \infty$ it would be a strict output growth targeting; or $\psi_3 \to \infty$ it would be exchange rate targeting. If ψ_1 is finite and $\psi_3 > 0$ a managed float is being implemented. ρ_R controls for the degree of NIR smoothing, which is an important variable for the conduct of monetary policy due to imperfect asset substitution, where $0 < \rho_R < 1$.

Due to the small scale model or a few endogenous pass through relation model, we are unable to expand this rule by including all influencing policy instruments (described in the section about Mongolian monetary policy) on these main policy variables: inflation, economic growth, and exchange rate. It is possible when we use a large-scale DSGE model consists of the enough auxiliary endogenous transmission relations on the variables.

Nominal exchange rate: We can introduce the NER policy by combining (16) and (17), which the later satisfies the relative PPP condition, as follows:

$$s_t = e_t + p_t^* - p_{h,t} \qquad \Rightarrow \qquad \Delta s_t = \Delta e_t + \pi_t^* - \pi_{h,t}$$

Then,

$$\begin{cases} \Delta s_t = \Delta e_t + \pi_t^* - \pi_{h,t} \\ \pi_t = \pi_{h,t} + \alpha \Delta s_t \end{cases} \qquad \Rightarrow \qquad \Delta s_t = \Delta e_t + \pi_t^* - \pi_t + \alpha \Delta s_t \Rightarrow$$

$$\Rightarrow e_t = e_{t-1} + \pi_t - (1-\alpha)\Delta s_t - \pi_t^* \tag{61}$$

where π_t^* is a world inflation shock which we treat as an unobservable.

The terms of trade (ToT): Instead of solving endogenously for the terms of trade, we add a law of motion for their growth rate to the system by the following $AR(1)$ process:

$$\Delta s_t = \rho_s \Delta s_{t-1} + \varepsilon_{s,t} \tag{62}$$

***Others*:** We assume that all other variables in the model, a_t, y_t^*, and π_t^*, will be determined exogenously by $AR(1)$ process, respectively.

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} \\ y_t^* &= \rho_{y^*} y_{t-1}^* + \varepsilon_{y^*,t} \\ \pi_t^* &= \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t} \end{aligned} \tag{63}$$

Equations from (58) to (63) form the linear rational expectation model which can be solved with standard techniques, for example, described in Sims (2001).

3.6.1 Equilibrium determinacy

This section is mainly based on the Herbst and Schorfheide (2016). Four structural equations, the DIS, the NKPC, the Taylor-type monetary policy rule, and the NER, and 5 exogenous $AR(1)$ processes, a_t, Δs_t, π_t^T, y_t^*, and π_t^*, form a LRE system that determines the evolution of

$$f_t = \left[y_t, \pi_t, r_t, e_t, \varepsilon_{R,t}, a_t, \Delta s_t, \pi_t^T, y_t^n, y_t^*, \pi_t^*\right] \tag{64}$$

In order to solve for the law of motion of f_t it is convenient to augment f_t by the expectations $E_t\{y_{t+1}\}$ and $E_t\{\pi_{t+1}\}$, defining the $n \times 1$ vector

$$\xi_t = \left[f_t', E\{y_{t+1}\}, E_t\{\pi_{t+1}\}\right]' \tag{65}$$

If we follow the solution method in Sims (2001), first we need transform the log-linear DSGE model into the canonical LRE form:

$$M_0 \xi_t = M_1 \xi_{t-1} + K\epsilon_t + X\delta_t \tag{66}$$

where $\epsilon_t = [\varepsilon_{\pi^*,t}, \varepsilon_{y^*,t}, \varepsilon_{\pi,t}, \varepsilon_{s,t}, \varepsilon_{a,t}, \varepsilon_{R,t}]'$. The vector δ_t captures one-step ahead rational expectations forecast errors. To write the equilibrium conditions of the model in the form of (66), we begin by replacing $E_t\{\Delta s_{t+1}\}$ and $E_t\{\Delta y_{t+1}^*\}$ with $\rho_s \Delta s_t$ and $\rho_{y^*} \Delta y_t^*$, respectively. We then note expectations errors for inflation and output as:

$$\begin{aligned} \delta_{y,t} &= y_t - E_{t-1}\{y_t\}, \\ \delta_{\pi,t} &= \pi_t - E_{t-1}\{\pi_t\} \end{aligned} \tag{67}$$

and define $\delta_t = [\delta_{y,t}, \delta_{\pi,t}]$. Using these definitions, the rational expectational log-linear model can be written as (66). The system matrices M_0, M_1, K, and X are functions of the DSGE model parameters θ.

Characterizations of a solution of this DSGE model is realized when the corresponding set of transversality conditions are satisfied. It implies that the law of motion should be non-explosive. This stability requirements restricts the set of solutions to (66). In general, the system have the following three possible solutions: i) no non-explosive (non-existence), ii) exactly one solution (uniqueness), and iii) many stable solutions (indeterminacy). The solution depends on the system matrices M_0, M_1, and K.

There are many alternative solution methods for the LRE systems and one of them is provided by Sims (2001). It shows that the LRE system can be transformed through a generalized complex Schur decomposition ($\mathcal{QZ}$) of M_0 and M_1, where Q, Z, Λ, and Ω are $n \times n$ matrices, such that $Q'\Lambda Z' = M_0$, $Q'\Omega Z' = M_1$, $QQ' = ZZ' = I$, and Λ and Ω are upper-triangular. Then, if we let $w_t = Z'\xi_t$ and pre-multiply (66) by Q to obtain:

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} (K\epsilon_t + X\delta_t) \tag{68}$$

The second set of equations can be written as:

$$w_{2,t} = \Lambda_{22}^{-1}\Omega_{22}w_{2,t-1} + \Lambda_{22}^{-1}Q_2 (K\epsilon_t + X\delta_t) \tag{69}$$

where $w_{2,t}$ is ordered by purely explosive $m \times 1$ vector ($0 \leq m \leq n$).

Then, if $w_{2,0} = 0$, the LRE system given in (66) has a non-explosive solution of ξ_t. It means that there is one can find a $k \times 1$ vector of rational expectations errors δ_t offsets the impact of $l \times 1$ vector of structural shock innovations ϵ_t on $w_{2,t}$:

$$\underbrace{Q_2K}_{m\times l}\underbrace{\epsilon_t}_{l\times 1} + \underbrace{Q_2X}_{m\times k}\underbrace{\delta_t}_{k\times 1} = \underbrace{0}_{m\times 1} \tag{70}$$

If $m = k$ and the matrix Q_2X is invertible, then the unique set of expectational errors that satisfy the stability of the system is given by

$$\delta_t = -\left(Q_2X\right)^{-1} Q_2K\epsilon_t$$

In general, it is not guaranteed that the vector δ_t need is uniquely determined by ϵ_t. An example of non-uniqueness (or indeterminacy) is the case in which the number of expectation errors k exceeds the number of explosive components m and (70) does not provide enough restrictions to uniquely determine the elements of δ_t. The set of non-explosive solutions (if it is non-empty) to the LRE system (66) can be obtained from $\xi_t = Zw_t$, (70).

In order to see how additional variables, the NER and the time-varying inflation target rate, influence to the equilibrium conditions, we summarize the different dimensions in the following Table 3.1.

Table 3.1: The equilibrium conditions of Sims approach

	n	m	l	k	
Benchmark	11	3	5	2	$\underbrace{Q_2K}_{3\times5}\underbrace{\epsilon_t}_{5\times1}+\underbrace{Q_2X}_{3\times2}\underbrace{\delta_t}_{2\times1}=\underbrace{0}_{3\times1}$
+ Exchange rate	12	4	5	2	$\underbrace{Q_2K}_{4\times5}\underbrace{\epsilon_t}_{5\times1}+\underbrace{Q_2X}_{4\times2}\underbrace{\delta_t}_{2\times1}=\underbrace{0}_{4\times1}$
+ Inflation target	12	3	6	2	$\underbrace{Q_2K}_{3\times6}\underbrace{\epsilon_t}_{6\times1}+\underbrace{Q_2X}_{3\times2}\underbrace{\delta_t}_{2\times1}=\underbrace{0}_{3\times1}$
+ ER & IT	13	4	6	2	$\underbrace{Q_2K}_{4\times6}\underbrace{\epsilon_t}_{6\times1}+\underbrace{Q_2X}_{4\times2}\underbrace{\delta_t}_{2\times1}=\underbrace{0}_{4\times1}$

In our case, m can be 4 at the maximum because y_t, π_t, r_t, and e_t are the aggregate macroeconomic variables, and these tend to be an non-stationary or an explosive. The number of expectations error is always 2, so $k = 2$. The dimensions of vector of structural shock innovations l depends on these additional variables are in the model or not. The benchmark model means that the using DSGE model is described without the exchange rate and the inflation target.

As we can see from the table, there is always possibility to have an non-explosive solutions because $m > k$ in every case. The corresponding numerical analysis of equilibrium will be performed in the next section by estimating the model.

3.6.2 Steady states

In this section, we describe the steady state relations and values of the main variables of the model which will be an important initial guess in the Bayesian estimation. In order to find these values, we use the assumptions when we made for building the model in the previous section.

The consumption Euler equation implies that the domestic NIR is $r = -\ln(\beta)$; thus, we can find $\beta = \exp(-r) = \exp(-0.17) = 0.84$ by using the average RIR of the observation period, which was approximately 17 percent quarterly. In here, we are following Lubik and Schorfheide (2007), in which parameterization is based on the terms of the steady state RIR.

We assumed that the model has the symmetric steady state satisfying the PPP condition $P_{h,t} = P_{f,t} = P$. Then, at the steady state, we have zero domestic and foreign goods inflation rates, $\pi_h = \pi_f^* = 0$. Moreover, we have $\mathcal{S} = 1$ or $s = 0$ since $\mathcal{S}_t = \dfrac{P_{f,t}}{P_{h,t}}$, which is given by equation (13).

Using the relationship between domestic and CPI inflation in equation (16), we can determine the steady state CPI inflation is also zero.

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t \qquad \Rightarrow \qquad \pi = 0$$

If we take a first-order difference from equation (17), we have $\Delta s_t = \Delta e_t + \pi_t^* - \pi_{h,t}$ and so, in the steady state, it will be $\Delta e = -\pi^*$. Since we are assuming that the foreign inflation dynamic is given as $AR(1)$, its steady state value would be zero, so $\Delta e = 0$. Then, the equation of UIP (26) determines the steady state foreign interest rate as equal to the domestic interest rate.

$$r_t - r_t^* = E_t\{\Delta e_{t+1}\} \qquad \Rightarrow \qquad r = r^*$$

According to the small open economy assumption $y = c$ which is used in transforming to log-linear form of aggregate demand and output. By the market equilibrium condition given by equation (41), the steady state foreign economy output equals to the domestic output.

$$y_t = y^* + \frac{1}{\sigma_\alpha} s_t, \quad s = 0 \qquad \Rightarrow \qquad y = y^*$$

Finally, we can determine y from the marginal cost condition given in equation (49) as follows,

$$mc_t = -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t$$

where, in the steady state, $mc = \ln\left(\frac{\varepsilon}{\varepsilon - 1}\right)$ and $\nu \equiv \ln\left(1 - \frac{1}{\varepsilon}\right)$. Then, we have

$$\begin{aligned}
\ln\left(\frac{\varepsilon}{\varepsilon-1}\right) &= -\ln\left(\frac{\varepsilon-1}{\varepsilon}\right) + (\sigma_\alpha + \varphi)\, y + (\sigma - \sigma_\alpha)\, y \\
\ln(1) &= (\sigma + \varphi)\, y \\
y &= \frac{0}{(\sigma + \varphi)} = 0
\end{aligned}$$

4 Estimation

In this section we present our estimation methodology and explain the results estimation of monetary policy rules. We also explain our observations of the data sets and how to choose prior distributions for the Bayesian analysis. At end of the section, we present the estimation results, its explanations and results of robustness analysis. The estimations are performed in the Dynare 4.4 with the Matlab 2015b by referring the code in Wieland et al. (2012), which is related to Lubik and Schorfheide (2007).

4.1 Methodology: Bayesian inference

We use Bayesian approach in estimation procedure because of the main purpose of this research that to estimate of the monetary policy rule (60) of the DSGE model of SOE.

The monetary policy rule parameters of the DSGE model are collected into the 4×1 vector $\psi = [\psi_1, \psi_2, \psi_3, \rho_R]$ and the non-policy parameters and the shock standard deviations are collected in the 12×1 vector θ. If we use the common assumption on structural shocks that is normal i.i.d (identically, independently distributed) we can have a joint probability distribution for the endogenous model variables. The vector of observables Y_t consists of annualized interest rates, annualized inflation rates, annualized inflation targets, output growth, nominal depreciation rates, and terms of trade changes.

$$Y_t = [4R_t, 4\pi_t, 4\pi_t^T, \Delta y_t, \Delta e_t, \Delta s_t]'$$

In the Bayesian approach, a prior distribution is determined with density $p(\psi, \theta) = p(\psi)p(\theta)$ on the structural parameters. The observed data set update the prior through the likelihood function of the DSGE model which is denoted by $\mathcal{L}_D\left(\psi, \theta | Y^T\right)$, where $Y^T = \{Y_1, Y_2, ..., Y_T\}$. Due to the Bayesian Theorem the posterior distribution of the parameters is given by:

$$p_D\left(\psi, \theta | Y^T\right) = \frac{\mathcal{L}_D\left(\psi, \theta | Y^T\right) p(\psi)p(\theta)}{\int \mathcal{L}_D\left(\psi, \theta | Y^T\right) p(\psi)p(\theta)d(\psi, \theta)} \tag{71}$$

Schorfheide (2000) and An and Schorfheide (2007) explain how Bayesian simulation technique generates posteriors. In general, the Bayesian estimation technique has benefits of that we can estimate all model parameters not only policy rule parameters. Moreover, the estimation approach can determine the dynamic properties of the DSGE model through impulse response functions and variance decompositions, thus we are possible to do some conclusions on the importance of structural shocks.

We are interested in the following two hypothesis. First, whether the BoM concern the inflation target announcement when they setting their monetary policy rule or not. It is given by equation (60) in which the NIR reacts to the inflation target rates and the deviation of total inflation from the inflation target. Second, whether the BoM react systematically to exchange rate movements or do not? In order to answer these hypothesis, we estimate a version $\mathcal{M}_1$ of the DSGE model in which the inflation target and the NER changes include in the monetary policy rule ($\psi_3 > 0$) and two different second version of $\mathcal{M}_0$ which expresses an alternative in each hypothesis. In other words, for the first hypothesis, a version $\mathcal{M}_0^1$ does not include the inflation target variable and for the second hypothesis, $\mathcal{M}_0^2$ is expressed when ψ_3 is restricted to be zero. Then, the posterior odds of each

$\mathcal{M}_0^j$ versus $\mathcal{M}_1$ are given by

$$\frac{\pi_{0,T}^j}{\pi_{1,T}} = \underbrace{\frac{\pi_{0,0}}{\pi_{1,0}}}_{Prior\ Odds} \cdot \underbrace{\frac{p\left(Y^T|\mathcal{M}_0^j\right)}{p\left(Y^T|\mathcal{M}_1\right)}}_{Bayes'\ factor}, \qquad j = 1, 2 \tag{72}$$

The first factor is the prior odds ratio to accept $\mathcal{M}_0^j$. The second term is called the Bayes' Factor and summarizes the sample evidence to accept $\mathcal{M}_0^j$ version. The term $p\left(Y^T|\mathcal{M}_i\right)$ is called marginal data density and appears as normalizing constant in the denominator of (71).

The logarithm of the marginal data density can be interpreted as maximized log-likelihood function penalized for model dimensionality. Under a $0-1$ loss function, the loss attached to choosing the wrong model is 1 and the optimal decision is to select the highest posterior model probability:

$$PO_{01}^j = \frac{p\left(Y^T|\mathcal{M}_0^j\right)\pi_{0,0}}{p\left(Y^T|\mathcal{M}_1\right)\pi_{1,0}} = \frac{p\left(\mathcal{M}_0^j|Y^T\right)}{p\left(\mathcal{M}_1|Y^T\right)}, \quad j = 1, 2$$

If we assume that we have two models, then

$$p\left(\mathcal{M}_0^j|Y^T\right) + p\left(\mathcal{M}_1|Y^T\right) = 1, \qquad j = 1, 2$$

Then,

$$p\left(\mathcal{M}_0^j|Y^T\right) = \frac{PO_{01}^j}{1 + PO_{01}^j}, \qquad p\left(\mathcal{M}_1|Y^T\right) = 1 - p\left(\mathcal{M}_0^j|Y^T\right)$$

4.2 Data description

We use observations on real output growth, inflation, NIRs, exchange rate changes, and terms of trade changes in our empirical analysis. All series, except of the inflation targets, are seasonally adjusted and at quarterly frequencies for the period $2000Q1$ to $2014Q3$ and are obtained from the BoM statistic database. Inflation target rates are observed from the annual Monetary Guidelines which are resolved from the Mongolian Parliament on the country's monetary policy between $2000-2014$.

Output growth rates are computed as log differences of real GDP and scaled by 100 to convert them into quarter-to-quarter percentages. Inflation rates are defined as log differences of the CPI and multiplied by 400 to obtain annualized percentage rates. The series of ToT are calculated by the ratio of price indices of exports and imports, and converted in log differences (scaled by 100) to obtain percentage changes in the terms of trade. The weighted average loan rates represent NIR and scaled by 400 to obtain annualized percentage rates. NER changes are defined as log differences of the nominal effective exchange rate index (NEER) and scaled by 100 to convert the indices into depreciation rates. All series are demeaned before estimation.

4.3 Choice of prior

As consistent with Lubik and Schorfheide (2007) we can divide all parameters in the model into three groups. First, theoretical structural parameters which do not depend on the country's characteristics: ψ_1, ψ_2, ψ_3, ρ_R, κ, τ, and ε^R. These coefficients are usually common in the related literatures. Second, country specific structural parameters: α, ρ_a, ρ_s, ρ_π, $\varepsilon_{a,t}$, $\varepsilon_{s,t}$, and $\varepsilon_{\pi,t}$. Third, structural parameters of the world economy: ρ_{y^*}, ρ_{π^*}, ε_{y^*}, and ε_{π^*}, which are also do not depend on the country's characteristics. The following Table 4.1 shows values of prior for Mongolia.

Assumptions and prior values of the theoretical structural parameters are same as in the article. These parameters are based on the common literatures related to Taylor-rule and the Phillips curve. The only change made in this group is that we increase the mean value of τ to 0.90 due to the assumption of unit substitution elasticity ($\sigma = \frac{1}{\tau}$) which will be used in the next chapter of the optimality analysis on the monetary policy. Moreover, in order to get the tight estimate we choose a relatively small standard deviation 0.05 on the prior distribution of τ.

We defined $\beta = 0.84$ in the previous section. The country specific parameter α, import share, is defined by the average import share of the observed period, which is about 60 percent. To specify ρ_s and $\varepsilon_{s,t}$, we estimate $AR(1)$ processes to growth rates of ToT, and obtain 0.94 and 0.10 respectively. These estimated parameters are little bit higher and tighter than the usual values, we assume that it centers at 0.90 with the standard error of 0.20, which allows it to vary widely. For the technological process, even we tried to obtain ρ_a and ε_a by estimating $AR(1)$ processes to the growth rate of Mongolian economy, we obtained a negative estimated value same as in the article for the UK and Australia; thus, we follow the article and choose the positive values in the article.

Table 4.1: Prior distributions for Mongolia

	Name	Domain	Density	Prior		Explanations
				P(1)	P(2)	
Theoretical parameters	ψ_1	$\mathbb{R}^+$	Gamma	1.54	0.50	from Lubik and Schorfheide (2007)
	ψ_2	$\mathbb{R}^+$	Gamma	0.25	0.13	
	ψ_3	$\mathbb{R}^+$	Gamma	0.25	0.13	
	ρ_R	$[0, 1)$	Beta	0.50	0.20	
	κ	$\mathbb{R}^+$	Gamma	0.50	0.25	
	ε^R	$\mathbb{R}^+$	InvGamma	0.50	4.00	
	τ	$[0, 1)$	Beta	0.90	0.05	Due to the assumption of an unit substitution elasticity
Country specific parameters	α	$[0, 1)$	Beta	0.60	0.20	An average import share of Mongolia during the observed period
	ρ_a	$[0, 1)$	Beta	0.20	0.10	from Lubik and Schorfheide (2007)
	ε_a	$\mathbb{R}^+$	InvGamma	1.00	4.00	
	ρ_s	$[0, 1)$	Beta	0.90	0.20	from $AR(1)$ processes on the Mongolian ToT and inflation target rates
	ε_s	$\mathbb{R}^+$	InvGamma	0.10	4.00	
	ρ_π	$[0, 1)$	Beta	0.97	0.05	
	ε_π	$\mathbb{R}^+$	InvGamma	0.21	4.00	
World economy's parameters	ρ_{y^*}	$[0, 1)$	Beta	0.97	0.05	from Lubik and Schorfheide (2007)
	ρ_{π^*}	$[0, 1)$	Beta	0.46	0.10	
	ε_{y^*}	$\mathbb{R}^+$	InvGamma	1.29	4.00	
	ε_{π^*}	$\mathbb{R}^+$	InvGamma	2.00	4.00	

Notes: P(1) and P(2) list the means and the standard deviations for beta, gamma, and normal distributions.

In order to input inflation target observations into the estimation process, we estimate $AR(1)$ processes to the seasonally adjusted quarterly inflation target values, which is built by dividing annual value into four equal parts. We obtained $\rho_{\pi^T} = 0.97$ and $\varepsilon_{\pi^T} = 0.21$.

In regarding to the world economy's parameters, we choose the estimated posterior values in the article. The article uses data between

1983 : $Q1$ and 2002 : $Q4$, so this is a pre-sampling period for our data period; thus the estimated posterior values can be a good representative prior values for our model.

4.4 Estimation results

The following Table 4.2 summarizes the Bayesian estimates of parameters of $\mathcal{M}_1$ and $\mathcal{M}_0^1$ models for Mongolia. In other words, these two models represent the cases when the BoM concern inflation target ($\mathcal{M}_1$) and when they do not concern it $\left(\mathcal{M}_0^1\right)$.

Table 4.2: Parameter estimation results of $\mathcal{M}_1$ and $\mathcal{M}_0^1$ models

	Prior		Posterior ($\mathcal{M}_1$)			Posterior $\left(\mathcal{M}_0^1\right)$		
	Mean	Std.dev	Mean	St.dev	90% HPD interval	Mean	St.dev	90% HPD interval
ψ_1	1.54	0.50	1.0636	0.19	[0.87 1.31]	0.9112	0.21	[0.57 1.22]
ψ_2	0.25	0.13	0.1764	0.09	[0.04 0.30]	0.1558	0.08	[0.05 0.27]
ψ_3	0.25	0.13	0.7048	0.16	[0.43 0.98]	0.6711	0.15	[0.44 0.89]
ρ_R	0.50	0.20	0.8862	0.02	[0.86 0.92]	0.8665	0.03	[0.83 0.91]
ε_R	0.50	4.00	0.6571	0.09	[0.51 0.79]	0.6694	0.08	[0.54 0.78]
κ	0.50	0.25	3.5937	0.27	[3.16 3.96]	3.6024	0.22	[3.22 3.96]
τ	0.90	0.05	0.8419	0.04	[0.77 0.91]	0.8432	0.05	[0.77 0.91]
α	0.60	0.20	0.8922	0.06	[0.81 0.97]	0.8787	0.06	[0.80 0.97]
ρ_a	0.20	0.10	0.7818	0.05	[0.69 0.87]	0.7803	0.04	[0.70 0.86]
ρ_s	0.90	0.20	0.1716	0.06	[0.06 0.26]	0.1657	0.06	[0.06 0.25]
ρ_π	0.97	0.05	0.9963	0.01	[0.99 1.00]	0.9959	0.003	[0.99 1.00]
ρ_{y^*}	0.97	0.05	0.8448	0.11	[0.65 1.00]	0.8282	0.14	[0.62 1.00]
ρ_{π^*}	0.46	0.10	0.3314	0.08	[0.20 0.44]	0.3400	0.07	[0.22 0.44]
ε_a	1.00	4.00	1.8149	0.44	[0.88 2.82]	1.6391	0.43	[0.85 2.59]
ε_s	0.10	4.00	12.2025	1.13	[10.04 14.23]	12.2839	0.95	[9.97 14.55]
ε_π	0.21	4.00	0.2185	0.02	[0.19 0.25]	0.2175	0.02	[0.18 0.25]
ε_{y^*}	1.29	4.00	36.2324	5.34	[17.69 53.24]	36.3648	11.63	[16.96 54.60]
ε_{π^*}	2.00	4.00	5.0084	0.59	[4.02 5.96]	4.9340	0.50	[4.09 5.74]

Notes: HPD - Highest Posterior Density

In here, the point estimates are the corresponding posterior means. The estimated results for two models are almost same, all parameters have a same sign and almost same standard deviations.

We use the results of $\mathcal{M}_1$ model for the explanations because this model includes all empirical variables that influence the NIR. Our findings mean that the BoM follows a moderately anti-inflationary policy, $\psi_1 = 1.0636$, and implements a concern for output, $\psi_2 = 0.1764$. The main interested parameter, ψ_3, is estimated as 0.7048 means that the bank relatively more concerns on the exchange rate movements when they implements interest-smoothing policy. There is also a reasonably high degree of interest-smoothing with an estimate of $\rho_R = 0.8862$. The preference parameter α is estimated as 0.8922 means that it is a higher than observable Mongolian import share.

The estimates of the stochastic processes shows that technology growth and inflation target rates have a relatively high degree of autocorrelations than in the prior means, $\rho_a = 0.7818$ and $\rho_\pi = 0.9963$ respectively. The rest of the stochastic processes have a smaller degree of autocorrelations, for instance the terms of trade processes has much smaller, $\rho_s = 0.1716$.

The influence of the individual shock is expressed by computing variance decompositions. Table 4.3 summarizes the results. In order to see short-term and long-term impacts, we compute it with conditional on different time horizons, 1 quarter, 1 year, 3 year, and many years. However, the most driving shock for each variables is same in the both horizons, and this is indicated as the same bolded shock impacts in each variable's column of the table. Thus, we use the long-term or final results of variance decompositions for the further explanations.

Table 4.3: Variance decompositions of $\mathcal{M}_1$ model, in percent

Variables / Shocks	Forecast horizon	Output	Inflation	Interest rate	Exchange rate
Monetary policy	$t = 1$ (1 quarter)	0.44	17.29	38.96	17.54
	$t = 4$ (1 year)	0.19	16.95	8.25	16.01
	$t = 12$ (3 year)	0.14	16.87	6.73	15.93
	$t = \infty$(final)	0.13	16.16	5.10	15.38
Terms of trade	$t = 1$ (1 quarter)	9.35	5.17	1.60	0.85
	$t = 4$ (1 year)	4.41	5.98	1.12	2.29
	$t = 12$ (3 year)	3.43	5.95	0.91	2.28
	$t = \infty$(final)	3.17	5.60	0.66	2.17
Technology	$t = 1$ (1 quarter)	1.91	**57.91**	**41.02**	**59.33**
	$t = 4$ (1 year)	0.81	57.48	85.40	55.34
	$t = 12$ (3 year)	0.60	57.60	86.79	55.45
	$t = \infty$(final)	0.54	**55.62**	**71.42**	**53.76**
Inflation target	$t = 1$ (1 quarter)	0.004	0.21	0.06	0.20
	$t = 4$ (1 year)	0.002	0.23	0.43	0.20
	$t = 12$ (3 year)	0.001	0.28	1.25	0.25
	$t = \infty$(final)	0.003	3.97	19.09	3.69
World output	$t = 1$ (1 quarter)	**87.88**	1.03	8.10	1.11
	$t = 4$ (1 year)	94.42	1.06	2.55	1.07
	$t = 12$ (3 year)	95.69	1.09	2.47	1.09
	$t = \infty$(final)	**96.04**	1.07	2.28	1.08
World inflation	$t = 1$ (1 quarter)	0.41	18.38	10.26	20.98
	$t = 4$ (1 year)	0.18	18.29	2.24	25.10
	$t = 12$ (3 year)	0.13	18.21	1.84	25.01
	$t = \infty$(final)	0.12	17.59	1.46	23.92

Notes: Table reports posterior means of variances based on the model $\mathcal{M}_1$. Bold means the highest contributions.

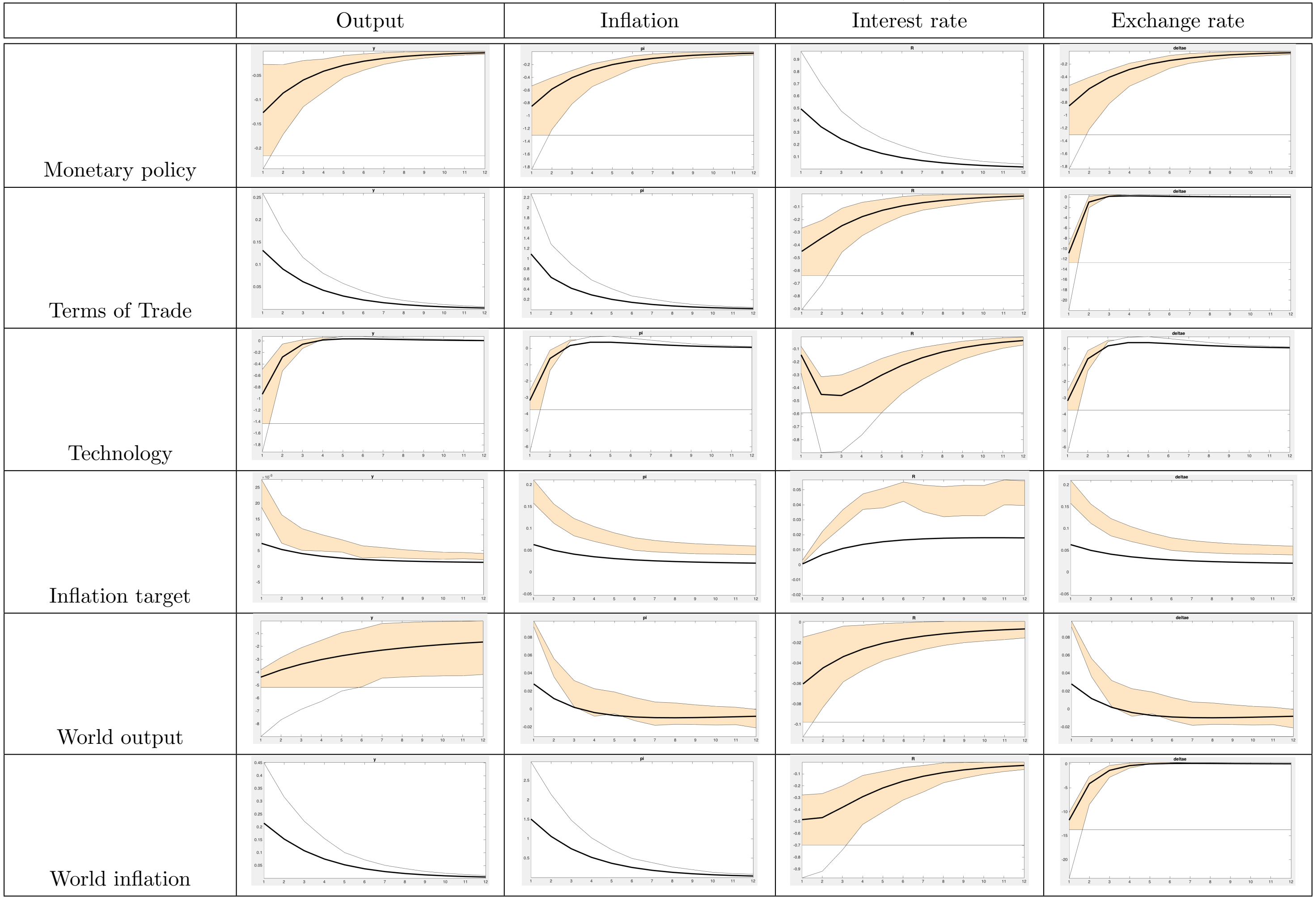
The only interesting results of comparison between different time horizon's impacts is relating to the shocks on inflation targeting rates. The influences of the shock are almost zero for all variables in the short-term but eventually increases in the long-term, for example, it explains only 0.06 percent of changes in interest rates in the short-term but in the long-term it will explain 19.09 percent of the changes. This result suggests that inflation target rates may have an influences on the long-term.

The changes in Mongolian GDP are almost fully, 96 percent, driven by the world output. This fact is consistent with the current Mongolian economic situation that the economic growth is highly depending on the foreign economies, in especially on the mining sector exports. The technology shock is the most influencing factor for the inflation, interest rate, and exchange rate change volatilities. The world inflation has a larger contribution than the monetary policy on the inflation (18 and 17 percent respectively) in long-term is likely the results of model misspecification as the the unobserved process including the effects of other foreign variables. Moreover, the world inflation shocks are the second driving factors for the exchange rate changes. If we follow Lubik and Schorfheide (2007) about the assumption on world inflation expression, which is π_t^* is interpreted as measurement error designed to capture deviations from PPP, then our model explains roughly about 21 percent (the difference between the world inflation and the ToT contribution percents) of Mongolian exchange rate movements.

In regarding with the ToT, it does not have a significant contribution to the domestic business cycles, between 0.7 and 5.6 percent, stands in consistent with the fact that ToT has a less than 10 percent explanatory power, for example Lubik and Teo (2005) which is mentioned in the article. As concluded in the article, the minor role of the ToT is not an undoubted results in international RBC literature, while some researchers prove that up to 50 percent of domestic GDP fluctuations to the ToT.

In order to describe the dynamic effects of the shocks, we compute impulse response functions, which are reported in Figure 4.1. The figure shows posterior means (thick lines) and 90% HPD intervals (tiny lines) for impulse responses of output, inflation, interest rate, and exchange rate changes to one-standard deviation structural shocks. We can see from these graphs which posterior mean is i) not signifiant when the 90% HPD intervals overlap, for example monetary shock on the interest rate, ii) strongly significant when the 90% percent HPD intervals include the posterior mean (most of them), and iii) weakly significant when the posterior

Figure 4.1: Impulse Responses of Mongolia ($\mathcal{M}_1$)



mean does not lie within the 90% HPD interval, for example inflation target rate shocks on the all variables.

An positive shock in the interest rate or contractionary monetary policy lowers output and inflation and appreciates the currency. In Mongolian economy, an improvement in the terms of trade (decreasing the domestic price) increases output and inflation level on impact via a nominal appreciation. The decline in the exchange rate prompts the BoM to decrease their policy rate which has an additional expansionary effect on output.

The technology is assumed as difference stationary innovations; thus, an positive technology shock should have an positive effect on production. However, we obtained an negative effects on output which is same as in the $AR(1)$ estimation on the Mongolian economic growth rates in when choosing the priors. For other variables, an positive technology shocks lower inflation and interest rates and thereby appreciate the currency. An positive shock in the inflation target would increase the output and inflation rates on impact via a lowering NIR. It means that the total effect of the inflation target terms in the monetary policy rule is an negative to the NIR, and a lower NIR will prompt to increase the output and so is inflation.

In regarding with the effect of rest of the world, we conclude that the world demand shocks would decrease output and interest rate in company with an increase in inflation and an exchange rate depreciation. Since world output shocks lower domestic potential output (equation (52)), we can see that the excess demand arises in equation (59), and as a result, inflation will be increased. By the monetary policy rule, these permanently increasing inflation leads central bank to raise NIR; however, on the other hand, an decreasing output lowers NIR due to this rule, so in Mongolian economy, the lowering effects dominate the increasing effect, and at the end the NIR decreases. An positive shock in world price inflation appreciate exchange rate (equation (61)) and raise inflation because the central bank reacts to this negative changes and to try to keep NIR without changes.

In the next, we answer two hypothesis described in the beginning of this section. We estimate two models, $\mathcal{M}_0^1$ and $\mathcal{M}_0^2$. In order to find answer we test the following two set of hypothesis by computing the posterior odds ratio, respectively. The results are reported in Table 4.4.

For the inflation target hypothesis, the marginal data density of the restricted model is 0.5683 smaller on a log-scale which translates into pos-

Table 4.4: Posterior odds

	Log marginal data densities		Odds
	$\mathcal{M}_0^j$	$\mathcal{M}_1$	
Inflation target hypothesis ($j = 1$)	-1044.84	-1045.41	1.7653
Exchange rate hypothesis ($j = 2$)	-1082.57	-1045.41	0.0000

Notes: The table reports posterior odds of the hypothesis H_0 vs H_1, assuming that the prior odds are one.

terior odds ratios of 1.7653. If we calculate the posterior model probability as described in the above, we have

$$p\left(\mathcal{M}_0^1|Y^T\right) = \frac{PO_{01}^1}{1 + PO_{01}^1} = \frac{1.7653}{1 + 1.7653} \approx 63.84\%$$

$$p\left(\mathcal{M}_1|Y^T\right) = 1 - p\left(\mathcal{M}_0^1|Y^T\right) \approx 36.16\%$$

The result says that the optimal model for the observation is $\mathcal{M}_0^1$ means that the BoM does not concern the inflation target rate when setting the nominal interest rate.

In case of the exchange rate hypothesis, the marginal data density of the model is 37.16 larger on a log-scale which translates into a posterior odds ratio of almost zero ($7e - 17$), and the corresponding posterior model probability is:

$$p\left(\mathcal{M}_0^2|Y^T\right) = \frac{PO_{01}^2}{1 + PO_{01}^2} = \frac{0.0000}{1 + 0.0000} \approx 0.00\%$$

$$p\left(\mathcal{M}_1|Y^T\right) = 1 - p\left(\mathcal{M}_0^2|Y^T\right) \approx 100.00\%$$

The result says that, in this case, the optimal model for the observation is $\mathcal{M}_1$ which is $\psi_3 > 0$. This leads us to conclude that the BoM pays very close attention to exchange rate movements when they are formulating their monetary policy in the Taylor-type rule.

4.5 Robustness

In general, there are two main approaches to robustness in the DSGE literature, i) to estimate in parallel a VAR (or a BVAR) and ii) to compare priors and posteriors within the DSGE model to assess mean and standard deviation, and overall reasonableness.

We use a second type of robustness approach based on the main restriction of the unit substitution elasticity assumption. We modified the prior on the elasticity due to the assumption; thus, we assess the robustness of the baseline results by relaxing the priors on τ. Since we chose $\tau = 0.90$ or a relatively high value in the estimation section, now we decrease this value to 0.80, 0.70, 0.50, and 0.30 and re-estimate the model on these alternative values of τ and all other priors are same as in the baseline model (Table 4.1). Table 4.5 provides information about the alternative priors and the resulted posteriors.

If we compare alternative estimates to the corresponding baseline estimates, we can see that the estimates of the τ are decreasing or shifted same direction in response to the prior mean changes. The estimated values of τs are close to the corresponding priors and sensitive to the changes in the prior mean. However, the differences in other policy parameter estimates are a relatively small; therefore, there would be no drastic changes in the conclusions based on the baseline posterior estimates.

Table 4.5: Alternative priors and posteriors for Mongolia

Name	Domain	Density	Prior mean (with st.dev 0.05)				
			Baseline	Alt. 1	Alt. 2	Alt. 3	Alt. 4
τ	$[0,1)$	Beta	0.90	0.80	0.70	0.50	0.30

Name	Posterior mean				
	Baseline	Alt. 1	Alt. 2	Alt. 3	Alt. 4
ψ_1	1.0636	1.1073	1.0660	1.0815	1.1900
ψ_2	0.1764	0.2088	0.1901	0.1689	0.1700
ψ_3	0.7048	0.7017	0.7133	0.7255	0.7562
ρ_R	0.8862	0.8884	0.8908	0.8847	0.8923
ε_R	0.6571	0.6638	0.6735	0.6703	0.6508
κ	3.5937	3.5774	3.6607	3.5682	3.5471
τ	**0.8419**	**0.7701**	**0.6903**	**0.4771**	**0.2700**
α	0.8922	0.8771	0.8751	0.8750	0.8725
ρ_a	0.7818	0.7949	0.8111	0.7919	0.7900
ρ_s	0.1716	0.1632	0.1602	0.1841	0.1700
ρ_π	0.9963	0.9966	0.9959	0.9949	0.9965
ρ_{y^*}	0.8448	0.9791	0.8299	0.7813	0.8122
ρ_{π^*}	0.3314	0.3287	0.3396	0.3284	0.3179
ε_a	1.8149	1.6514	1.4951	1.6096	1.7331
ε_s	12.2025	12.1504	12.12428	12.0422	12.1314
ε_π	0.2185	0.2150	0.2153	0.2144	0.2237
ε_{y^*}	36.2324	23.5542	14.1405	5.7761	2.3431
ε_{π^*}	5.0084	5.0693	5.0563	5.0994	4.9581

5 Conclusion

In this essay, we estimate the modified small-scale DSGE of SOE setting using Bayesian methods for the Mongolian data. In order to answer to proposed hypothesis, we modified a generic Taylor-rule to one that consistent with the current Mongolian monetary policy regime.

Our main conclusion is that the BoM do not concern the time-varying inflation target rates on its policy rates and the BoM responds to exchange rate movements systematically. Our findings suggest that Mongolia is a managed flexible exchange rate regime country and the CPI inflation-based Taylor rule (CITR, for short) forms the current effective policy rule. Moreover, the shocks of the ToT do not have a significant contribution to the business cycle and stands in consistent with the fact that the ToT has a less than 10 percent explanatory power.

As consistent with Lubik and Schorfheide (2007), we agree that our used model may be misspecified because of the lack of imperfect pass-through of NER changes into domestic import prices and our assumption of exogenous ToT movements. Moreover, our finding that the ToT has almost negligible influence in the output is a conflicted result with studies based on VAR, in particular, calibration studies. The model has a weak endogenous transmission mechanism on the ToT; thus, introducing additional dynamics through capital accumulation, different production sectors and internationally incomplete asset markets would prove that the ToT's different character.

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