

## Noisy Point Clouds Registration Using FFT Based on Multi-Stage Noise Removal

Byambajargal N.<sup>1\*</sup>, Ankhbayar B.<sup>2</sup>, Oyundolgor Kh.<sup>1</sup> and Enkhbayar A.<sup>2\*</sup>

<sup>1</sup>Department of Information and Computer Sciences, School of Engineering and Applied Sciences, Ulaanbaatar, Mongolia

<sup>2</sup>Department of Applied Mathematics, School of Engineering and Applied Sciences, Ulaanbaatar, Mongolia

\*enkhbayar.a@seas.num.edu.mn

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### Abstract

In this paper, we introduce a multi-stage fine registration technique for registering noisy point clouds. At each stage, discrete surfaces that overlap each other are simultaneously transformed into a frequency domain by a fast Fourier transform (FFT) algorithm. In the frequency domain, an adjustable function is used as the low-pass filter, and then discrete surfaces are reconstructed by an inverse Fourier transform. The iterative closest point algorithm is used to register the newly generated surfaces and obtain the registration parameters. We then registered the original point clouds by using these parameters. The next stages are implemented in the same way as in the above; only the parameters are changed in the filter. After a few stages, our method can give a better result for the registration of noisy point clouds. We experimented with the proposed method for registering many types of noisy point clouds such as noisy point clouds with different noise levels or noisy and sparse point sets.

**Key words:** Noisy point clouds, fast Fourier transform, discrete surfaces, ICP algorithm

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### 1 Introduction

One of the important problems for reconstructing a three-dimensional (3D) model is the registration of the point clouds captured by scanning devices. Because of the complexity of an object, occlusion, self-occlusion, and the limitation of scanning devices, multiple scans are typically required to capture all surfaces of an object. To reconstruct a 3D model, we need to register the separate point clouds obtained from different scans into a common coordinate system.

In the last two decades, many methods have been developed to register point clouds (1), (2), (3), (4), (5). Most of the currently existing methods can be classified into two categories: rough registration and fine registration. Rough registration algorithms generate an initial alignment between two point clouds.

Various techniques are employed to generate the initial alignment of pairwise registration such as tracking the position of the scanner, placing additional markers, computing the local and global features, or using some external information. In

recent years, many local feature-based methods are commonly used for the initial alignment of point clouds (6).

For example, Guo et al. (7) proposed a rotational projection statistics (RoPS)-based feature descriptor, and Rusu et al. (8) presented point feature histograms as multidimensional features for registering point clouds. Recently, Yang et al. (9) proposed a novel feature descriptor called local feature statistics histogram, whereas Mellado et al. (10) introduced a growing least squares-based descriptor for the rough registration of point clouds.

In the second category, fine registration, the algorithms obtain the most accurate solution of the point clouds compared to the solution of those that have been roughly registered. The most popular fine registration method in practice is the Iterative Closest Point (ICP) algorithm (11), (12). The ICP algorithm computes the rigid transformation that minimizes the sum of the squared distances between two point clouds. The process is iterated until some convergence criteria are met. The ICP algorithm converges well under some conditions such as point clouds with a

good initial alignment, those with low noise, and those with similar resolutions. However, in the other cases, the ICP algorithm converges to a local minimum and fails to register the point clouds. To improve the ICP algorithm, other studies have proposed many variants and extensions (3), (13), (14), (15).

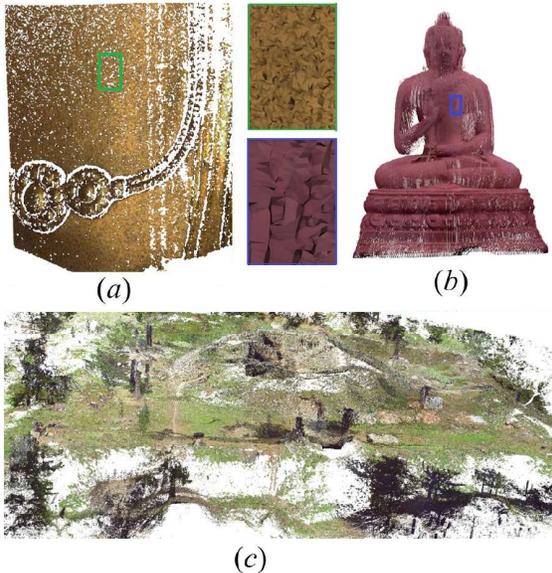


Fig. 1. Noisy point clouds were obtained by different scanners from various objects. The magnified sections show polygon meshes of point clouds.

The existence of noise in point cloud data is a challenging issue, not only in the ICP algorithm, but also in feature-based registration algorithms because some feature-based descriptors such as feature lines are unstable on noisy point clouds.

In practice, a point cloud captured by scanning devices includes some noise because of the limitation of such devices or the physical property of an object. For example, Fig. 1 shows some noisy point clouds. The part of a point cloud of a gilt-bronze statue scanned by a NextEngine 3D laser scanner is shown in Fig. 1(a). Because of surface shininess, the

laser is reflected too much, causing what is known as noise. Figure 1(b) shows a point cloud of a small clay statue obtained by a Matter and Form 3D scanner. The presence of strong noise in the point cloud, however, can be observed owing to the limitation of the scanner. In Fig. 1(c), a point cloud of grassland and herbage captured by a FARO laser scanner is shown. Because of the roughness of the land surface, there were some noises in the point cloud too. Therefore, a robust point cloud registration method for registering noisy point clouds is very important in practical applications. Taubin

(21) introduced polyhedral surface fairing based on signal processing techniques. He defined polyhedral surface smoothing by generalizing the discrete Fourier analysis to 2D discrete surface signals and reduced the surface smoothing problem to low-pass filtering. For mesh filtering, eigenvectors of the Laplacian operator are computed explicitly in the spatial domain via convolution. This iterative method, however, requires a high computational cost. In addition, Pauly and Gross (22) presented a method for processing point-sampled objects using a spectral method. In their method, point-sampled models are split into a number of patches, each patch is resampled geometrically, and low-pass filtering is conducted in the frequency domain by Fourier analysis such as FFT.

Our main contributions are as follows:

- A single-iteration algorithm for removing noise from point clouds using FFT and low-pass filtering in the frequency domain;
- A new and robust algorithm for registering noisy point clouds using adjustable filter functions.

## 2 Related works

Currently, many robust algorithms have been proposed for registering noisy point clouds. Some approaches improved the classical ICP algorithms, whereas others introduced a stable feature descriptor for noisy point sets.

Segal et al. (15) introduced a generalization of the ICP algorithm that performs plane-to-plane matching by combining the “point-to-point” and “point-to-plane” ICP algorithms into a single probabilistic framework. Their method is based on attaching a probabilistic model to the minimization process so that the surface information from both point clouds can be incorporated easily into the optimization algorithm. This is a robust approach for registration. Jian and Vemuri (23) proposed a similar approach based on a probabilistic modeling framework for registering noisy point clouds.

Gelfand et al. (13) proposed a point selection strategy to improve the registration performance of the ICP algorithm. They presented a method for detecting uncertainty in pose and minimized this uncertainty by choosing samples that constrain potentially unstable transformations. Meanwhile, Granger et al. (24) formulated point cloud registration as a maximum likelihood problem and presented a new variant of the ICP algorithm by applying the expectation maximization principle.

Fitzgibbon (25) introduced a variant of the ICP algorithm using matching points to the minimization strategy. The registration error is directly minimized using a generalpurpose nonlinear optimization algorithm.

Recently, Yang et al. (26) proposed a globally optimal algorithm for registering point clouds under the  $L_2$  error metric defined in the ICP algorithm. The method is based on a branch-and-bound scheme that searches the entire 3D motion space ( $SE(3)$ ). The authors derived novel upper and lower bounds for the registration error function by exploiting the special structure of  $SE(3)$ .

On the other hand, some authors proposed stable descriptors for registering noisy point clouds. For example, Zhong (27) introduced a new 3D shape descriptor called intrinsic shape signature (ISS) to characterize a local region of a point cloud. ISS uses the eigenvectors of the covariance matrix of a point to describe its neighbors. The algorithm gives a good result on noisy point sets.

Yang et al. (9) proposed a novel feature descriptor called local feature statistics histogram (LFSH) for the robust registration of point clouds. By combining a set of low-dimensional geometric features, LFSH incurs a minimal loss in the local shape descriptions, but it is robust to noisy and varying point cloud resolutions. Guo et al. (7) proposed a RoPS-based feature descriptor that is based on an accurate and robust algorithm for registering point clouds. Boughorbel et al. (28) presented a robust registration method based on Gaussian fields. They presented a Gaussian field criterion that consists of a Gaussian mixture, depending on the distance and point attributes such as the local shape descriptors. Later, Boughorbel et al. (29) extended the work in (28) by using a continuously differentiable energy function. The method (29) shows robustness in the presence of strong noise.

Amamra et al. (30) presented a recursive robust filtering method for feature-based point cloud registration. The algorithm is based on a recursive optimal state estimation. The registration problem was fitted to the Kalman filter scheme, and it was robust to uncertainties caused by noisy feature localization. Deng et al. (31) proposed a local feature

descriptor-based point pair local topology. The topology descriptor is defined by a histogram that is constructed using the weighting of the distance measures and angle measures based on a local point pair.

Sandhu et al. (32) presented a particle filtering approach for registering point clouds. In their work, stochastic motion dynamics are introduced to widen the narrow band of convergence which is used to tackle the registration task. The method works well on point clouds with poor initialization and noise without any geometric assumption on the point cloud density.

### 3 Two-dimensional discrete Fourier transform

Let  $f(x; y)$  be a discrete function defined on  $N \times M$  point sets. The discrete Fourier transform (DFT)  $F(u; v)$  of a function  $f(x; y)$  is given by the following equation:

$$F(u, v) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) e^{-j2\pi(\frac{ux}{N} + \frac{vy}{M})} \quad (1)$$

Similarly,  $f(x, y)$  is obtained by the inverse discrete Fourier transform (IDFT) for the given  $F(u, v)$  by the following equation:

$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} F(u, v) e^{-j2\pi(\frac{ux}{N} + \frac{vy}{M})}. \quad (2)$$

The DFT of a discrete surface or its IDFT can be accelerated by a FFT. A Fourier transform converts a signal from its original domain (often a time or space domain) to a representation in the frequency domain. The Fourier spectrum  $|F(u, v)|$  is defined as

$$|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{1/2}, \quad (3)$$

where  $R(u, v)$  and  $I(u, v)$  are the real and imaginary parts of  $F(u, v)$ , respectively (33). The Fourier spectrum reveals the frequency components in the input data.

In the frequency domain, the high-frequency components can be eliminated by a low-pass filtering operation as follows:

$$W(u, v) = |F(u, v)|H(u, v), \quad (4)$$

where  $H(u, v)$  is a low-pass filter function and  $W(u, v)$  is the power spectrum where the high-frequency components are eliminated. In signal processing, there are many low-pass filter functions such as Butterworth filter, Chebyshev filter and Gaussian filter.

A frequency-domain filtering operation can be represented in a spatial domain as follows:

$$w(x, y) = f(x, y) * h(x, y), \quad (5)$$

where  $h(x, y)$  is the IDFT of  $H(u, v)$ ,  $w(x, y)$  is a smoothing function, and  $*$  is a convolution operator.

#### 4 Noisy point cloud registration

The input for our algorithm is two partially overlapped noisy 3D point clouds,  $P = \{\mathbf{p}_i \in R^3\}$  and  $Q = \{\mathbf{q}_i \in R^3\}$ , obtained from a scanning device. Let  $P_i$  and  $Q_i$  be point sets on the overlapped area of the point clouds, and let  $P$  and  $Q$  be their explicit discrete surfaces, respectively. Let  $\bar{d}_P$  and  $\bar{d}_Q$  be the average distance between points in point sets  $P_i$  and  $Q_i$ , respectively.

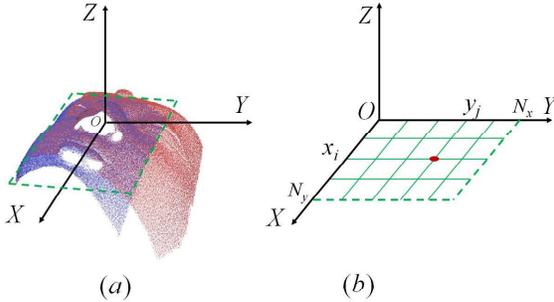


Fig. 2. (a) Two partially overlapped noisy point clouds and the minimum bounding rectangle; (b) 2D grid generation.

##### 4.1 Point cloud resampling

To use a discrete Fourier transform, we resample point sets  $P_i$  and  $Q_i$ . The first stage of resampling is to construct a local coordinate system for point sets  $P_i$  and  $Q_i$ . Therefore, the covariance matrix  $C$  is defined from the union of two point sets,  $PI$  and  $QI$ , and denoted by  $PI \cup QI$ . If  $\mathbf{v}_j$ , ( $j = 1, \dots, K$ ) are points of the set  $P_i \cup Q_i$ , then  $C$  can be defined as

$$C = \frac{1}{K} \sum_{i=1}^K (\mathbf{v}_i - \bar{\mathbf{v}})^T (\mathbf{v}_i - \bar{\mathbf{v}}), \quad (6)$$

where  $\bar{\mathbf{v}}$  is the average of the set,  $\bar{\mathbf{v}} = \frac{1}{K} \sum_{i=1}^K \mathbf{v}_i$ .

Let  $\lambda_1, \lambda_2$ , and  $\lambda_3$  ( $\lambda_1 \leq \lambda_2 \leq \lambda_3$ ) be the eigenvalues of matrix  $C$ , and let  $\mathbf{e}_1, \mathbf{e}_2$ , and  $\mathbf{e}_3$  be the corresponding eigenvectors. In principal component analysis, the eigenvector  $\mathbf{e}_1$  to the smallest eigenvalue  $\lambda_1$  estimates the normal vector of point  $\bar{\mathbf{v}}$  (?) and a local coordinate system ( $OXYZ$ ) can be constructed by the eigenvectors  $\mathbf{e}_1, \mathbf{e}_2$ , and  $\mathbf{e}_3$ . The  $OZ$  axis is selected along the normal vector of point  $\bar{\mathbf{v}}$ .

In this coordinate system, a minimum bounding rectangle of the overlapping area of surfaces  $P$  and  $Q$  is established on the  $OXY$  plane. For example, Fig. 2(a) shows two noisy point clouds obtained from the face of a statue. Both point clouds contained strong noise and partially overlapped surfaces produced by the rough registration. The minimum bounding rectangle of the overlapping area is drawn by green lines.

Consequently, the bounding box is translated into the first quadrant of the coordinate plane  $OXY$  and a uniform 2D grid is constructed on the bounding rectangle. The grid size  $N_x \times N_y$  and grid step size are chosen to be smaller than  $\min(\bar{d}_P, \bar{d}_Q)$ . In the constructed grid, the discrete surfaces  $P$  and  $Q$  are resampled into grid spaces. For example, the resampling values  $fP(x_i, y_j)$  ( $i = 0, 1, \dots, N_x - 1, j = 0, 1, \dots, N_y - 1$ ) of point cloud  $P$  are computed by the following rules:

- For each grid at point  $(x_i, y_j)$ , only four neighbors are selected from point cloud  $P$  as

$$\bar{p}_1 = \{p_l | \min(d((x_i, y_j), (x_{p_l}, y_{p_l}))), x_{p_l} \leq x_i \text{ and } y_{p_l} \leq y_j, \forall p_l \in P\}, \quad (7a)$$

$$\bar{p}_2 = \{p_l | \min(d((x_i, y_j), (x_{p_l}, y_{p_l}))), x_{p_l} \geq x_i \text{ and } y_{p_l} \leq y_j, \forall p_l \in P\}, \quad (7b)$$

$$\bar{p}_3 = \{p_l | \min(d((x_i, y_j), (x_{p_l}, y_{p_l}))), x_{p_l} \leq x_i \text{ and } y_{p_l} \geq y_j, \forall p_l \in P\}, \quad (7c)$$

$$\bar{p}_4 = \{p_l | \min(d((x_i, y_j), (x_{p_l}, y_{p_l}))), x_{p_l} \geq x_i \text{ and } y_{p_l} \geq y_j, \forall p_l \in P\}, \quad (7d)$$

where  $d((x_i, y_j); (x_{p_l}, y_{p_l}))$  is the distance between points  $(x_i, y_j)$  and point  $(x_{p_l}, y_{p_l})$ , and  $x_{p_l}$  and  $y_{p_l}$  are respectively the first and the second coordinate of point  $p_l$ . By Eq. (7), the four closest points to the grid point  $(x_i, y_j)$  are found.

- The value of  $fP(x_i, y_j)$  is computed from points  $\bar{p}_1, \bar{p}_2, \bar{p}_3$ , and  $\bar{p}_4$  by inverse distance weighting interpolation as

$$fP(x_i, y_j) = \sum_{l=1}^4 \frac{w_l z_{p_l}}{\sum_{r=1}^4 w_r}, \quad (8)$$

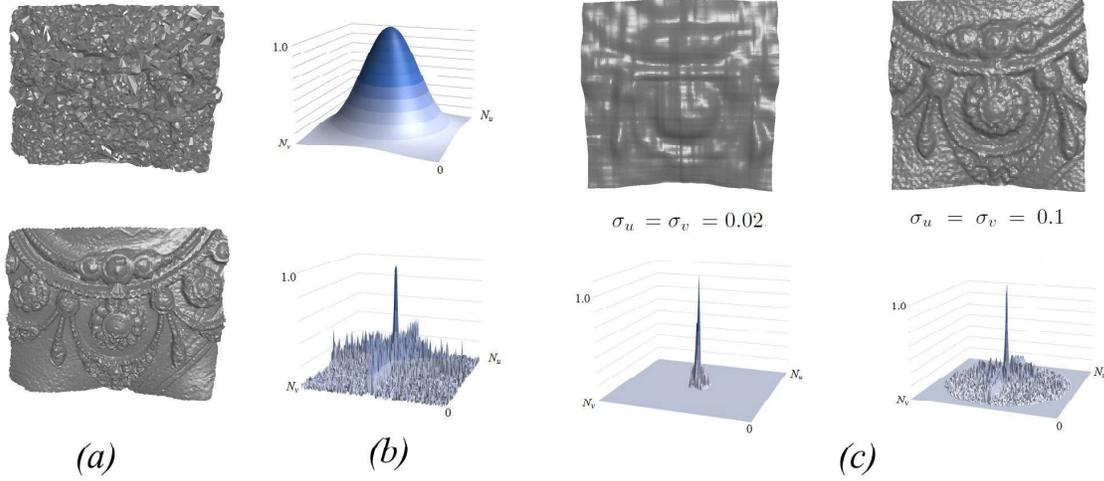


Fig. 3. (a) Point cloud with strong noise and an image of the original model; (b) Graph of the Gaussian function and Fourier spectrum of the point cloud; (c) The filtered Fourier spectra and the corresponding surfaces at various parameters

where

$$w_l = \frac{1}{d((x_i, y_j), (x_{\bar{p}_l}, y_{\bar{p}_l}))} \quad (9)$$

is the inverse distance weighting function, and  $z_{\bar{p}_l}$  is the third coordinate of point  $\bar{p}_l$ .

The resampling values  $f_Q(x_i, y_j)$  ( $i = 0, 1, \dots, N_x - 1, j = 0, 1, \dots, N_y - 1$ ) of point cloud  $Q$  are computed in the same way as  $f_P(x_i, y_j)$  by Eqs. (7)-(9) on the same grid. Figure 2(b) shows the uniform 2D grid constructed in the bounding rectangle.

#### 4.2 Noise removal in the frequency domain

Two resampled surfaces,  $f_P(x_i, y_j)$  and  $f_Q(x_i, y_j)$ , on the uniform grid can be transformed into the frequency domain by Eq. (1). To take advantage of FFT, we should make grid size  $N_x$  and  $N_y$  a power of 2. The Fourier spectra of  $f_P(u_p, v_j)$  and  $f_Q(u_p, v_j)$  are computed using Eq.(3), respectively.

To eliminate the high-frequency components of  $|f_P(u_p, v_j)|$  and  $|f_Q(u_p, v_j)|$ , adjustable window function that is zero-valued outside of some chosen interval can be used. In typical applications, the window functions used are non-negative, smooth, bell-shaped surfaces such as Gaussian function, generalized normal, Dolph-Chebyshev function and Tukey function. In our research, we selected a Gaussian window function as a filter:

$$G(u_i, v_j) = e^{-\left(\frac{u_i^2}{2\sigma_u^2} + \frac{v_j^2}{2\sigma_v^2}\right)}, \quad (10)$$

where  $\sigma_u$  and  $\sigma_v$  are the parameters of the Gaussian function. By changing the  $\sigma_u$  and  $\sigma_v$  parameters, we can change the shape of the function; in order words, a filter function can be adjusted by the parameters.

A low-pass filter function in the frequency domain is formulated as follows:

$$H(u_i, v_j) = \begin{cases} 1, & \text{if } |F(u_i, v_j)| \leq G(u_i, v_j) \\ 0, & \text{if } |F(u_i, v_j)| > G(u_i, v_j). \end{cases} \quad (11)$$

The high-frequency components of the Fourier spectrum are removed using this filter function in Eq. (4).

In the next step, a smooth discrete surface is constructed easily by transforming the filtered frequency data by IFFT in Eq. (2). In Fig. 3(a), a noisy point cloud bounded by the minimum rectangle and an image of the model constructed by a noiseless point cloud are shown. Figure 3(b) shows a Gaussian function for the filtering and the normalized Fourier spectrum of the noisy point cloud, whereas Fig. 3(c) shows the constructed smooth surfaces with the corresponding filtered spectra at various parameters of  $\sigma$  in a Gaussian function.

Once the smooth surfaces  $\bar{f}_P(x_p, y_j)$  and  $\bar{f}_Q(x_p, y_j)$  ( $i = 0, 1, \dots, N_x - 1, j = 0, 1, \dots, N_y - 1$ ) are constructed on the grid, the surface points are pruned on the basis of the distance-based criteria. We formulated the criteria as follows: if the surface point  $\bar{f}_P(x_p, y_j)$  defined on the grid satisfies the following condition, then we assume that the point  $\bar{f}_P(x_p, y_j)$  belongs to the overlapping surface  $P_j$

$$\min(d(\bar{f}_P(x_i, y_j), \bar{p}_l), l = 1, 2, 3, 4) < \min(\bar{d}_P, \bar{d}_Q) \quad (12)$$

where,  $d(\bar{f}_P(x_p, y_j), \bar{p}_l)$  is the distance between points  $\bar{f}_P(x_p, y_j)$ ,  $\bar{p}_l$  is defined by Eq. (7), and  $\min(\bar{d}_P, \bar{d}_Q)$  is the average distance of input point clouds  $P$  and  $Q$ .

#### 4.3 Point cloud registration

We can obtain two smooth surfaces,  $\bar{P}_1$  and  $\bar{Q}_1$ ,

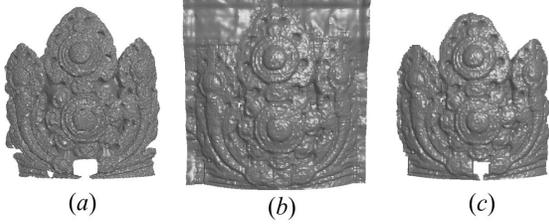


Fig. 4. (a) Noisy point cloud; (b) smoothed surface on the grid; (c) result after pruning the points.

overlapping each other by applying the techniques described in the previous sections. These point clouds can be registered by using the ICP algorithm (12), which uses the point-to-plane error metric:

$$E = \sum_i [(R\mathbf{p}_i + T - \mathbf{q}_i) \cdot \mathbf{n}_i]^2 \quad (13)$$

where  $R$  is a rotation matrix,  $T$  is a transformation matrix, and  $(p_i, q_i)$  are point pairs with normal  $\mathbf{n}_i$ . By implementing the algorithm in Eq. (13), we can obtain six parameters including the operations of the rotation and the translation.

In the final step, the original noisy point clouds can be registered by the parameters obtained by the ICP algorithm. However, depending on the noise level and the rough registration of the point clouds, the proposed technique will require a multi-stage implementation. For each stage, the same techniques such as resampling of point clouds, filtering of noise, and registration of a point cloud by the ICP algorithm are implemented. The only difference between the stages is the parameter value for the filtering function. For example, Fig. 5 shows the process for registering noisy point clouds. Figure 5(a) shows a point cloud with a strong noise and its initial alignments. For the registration of these point clouds, three stages are required. Figure 5(b)-(d) show point clouds whose noises were removed by the different parameter values of a Gaussian function in Eq. (10) and the registration result of noisy point clouds in each step. The parameter values such as  $\sigma_{u,1} = \sigma_{v,1} = 0.02$ ,  $\sigma_{u,2} = \sigma_{v,2} = 0.05$  and  $\sigma_{u,3} = \sigma_{v,3} = 0.1$  are defined by the user. In the first stage, all noises and some small features are removed from the point clouds. Thus, the result of the stage is an improved rough alignment of the point clouds. Starting from the second and the third stages, the fine registration is performed between point clouds with certain features and a small level of noise.

A number of stages of the proposed method depend on the noise level of the point clouds and on the complexity of the original model. The registration process will continue until the average variation of the point clouds is stabilized. For the noisy point clouds, a distance-based registration

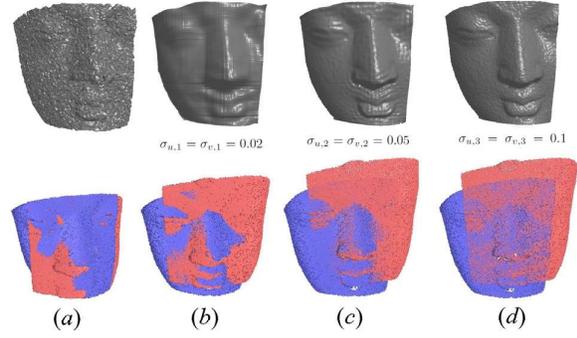
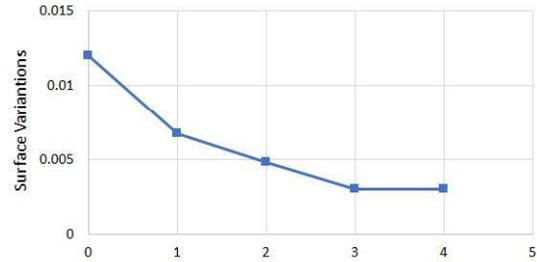


Fig. 5. (a) Original noisy point cloud and rough registration of point clouds; (b)-(d) resampled surfaces with different parameter values of  $\sigma$  and point clouds registration results at each stage.

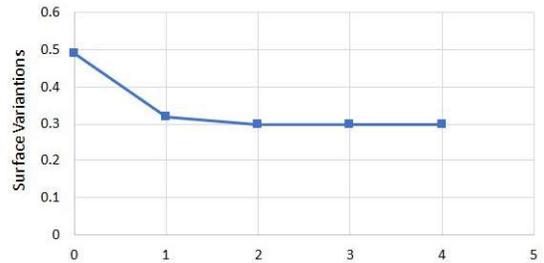
error is not efficient. Therefore, we considered an error measurement based on the average variation value of the registered point clouds. For the computation of the variation value for each point in the point cloud  $P_i \cup Q_i$ , the covariance matrix  $C$  is defined from each point and its closest  $K$  neighbors by Eq. (6) and the variation of point  $p$  is defined by the following function proposed by Pauly et al. (?):

$$\sigma(\mathbf{p}) = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2},$$

where  $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$  are the eigenvalues of the covariance matrix  $C$  with  $\lambda_0 \leq \lambda_1 \leq \lambda_2$ .



(a)



(b)

Fig. 6. (a) Average variation; (b) average distance at each step of the proposed method. Fig 6.jpg

Figure 6 shows graphs of the average variation value of the point cloud  $P_i \cup Q_i$ , and the average nearest distance value between point clouds  $P_i$  and  $Q_i$  at each step of the proposed method's process shown in Fig. 5.

The process for registering noisy point clouds is summarized in Algorithm 1.

1.

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**Algorithm 1.** Algorithm for registering noisy point clouds,

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**Input:** Noisy point clouds  $P$  and  $Q$

**Input:**  $\sigma_i$  parameters of the Gaussian function

1. **do**
  2. define overlapping areas  $P_i$  and  $Q_i$
  3. get next  $\sigma_i$
  4. construct the coordinate system  $OXY Z =$  for  $P_i \cup Q_i$
  5. define the uniform grid with a size of  $N_x \times N_y$
  6. **for each**  $x_i$  and  $y_j$
  7. compute the resampling values  $fP(x_i, y_j)$  and  $fQ(x_i, y_j)$
  8. **end for**
  9. compute  $FFT(fP(x_i, y_j))$  and  $FFT(fQ(x_i, y_j))$
  10. filter  $|FP(u, v)|$  and  $|FQ(u, v)|$  by using a Gaussian function
  11. compute  $FFT(fP(u, v_j))$  and  $IFFT(FQ(ui, vj))$   
//  $\tilde{fP}(x_i, y_j)$  and  $\tilde{fQ}(x_i, y_j)$  are obtained
  12. implement the ICP algorithm on  $\tilde{fP}(x_i, y_j)$  and  $\tilde{fQ}(x_i, y_j)$   
// 6 parameters for registration are obtained
  13. 13: register point clouds  $P$  and  $Q$  by parameters
  14. **while** average variation not stabilized
- 

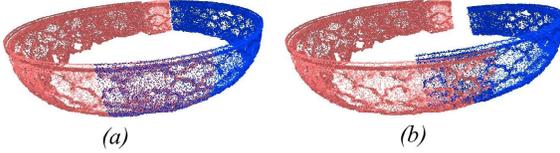


Fig. 7. (a) Result of the proposed algorithm; (b) failed registration of the point clouds by the ICP algorithm.

## 5 Experimental results

We applied the proposed method for registering different types of point clouds such as point clouds with strong noise or noisy point clouds with different resolutions or different amounts of noise. The experiments were performed on a machine with a 2.8-GHz Intel Xeon E5-1603 processor and 8GB of RAM.

In our experiment, the implementation was done for different types of point clouds such as those with

noise owing to the limitation of scanning devices or to the physical property of the objects. For the others, noises were created by adding Gaussian noise with zero mean and a variance of a certain percentage of the average distance of the point clouds.

For example, approximately 90% of noise was added to the point clouds in the experiment shown in Fig. 3. Rough registration was manually implemented by selecting the feature points. In the first experiment, we registered the point clouds of an object with a round shape. This type of object is good for showing the robustness of the method. To the point clouds, approximately 85% of noise was added. The proposed method requires two stages for registering point clouds. The final result is shown in Fig. 7(a). Figure 7(b) shows the result of the ICP algorithm that failed to register these point clouds.

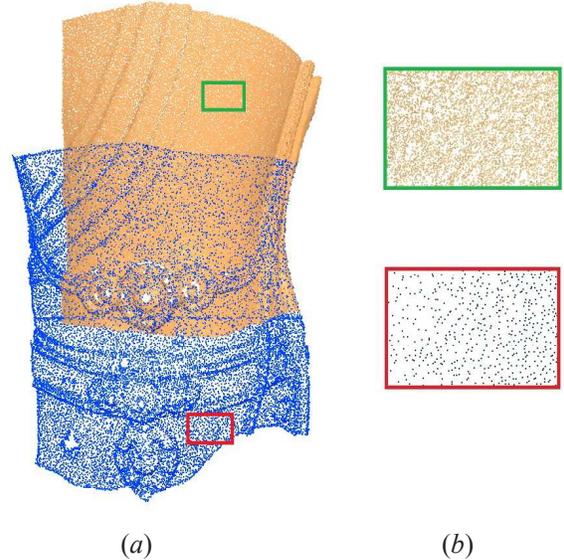


Fig. 8. (a) Result of the proposed algorithm for registering noisy point clouds with different densities; (b) the magnified sections of a point cloud with different densities

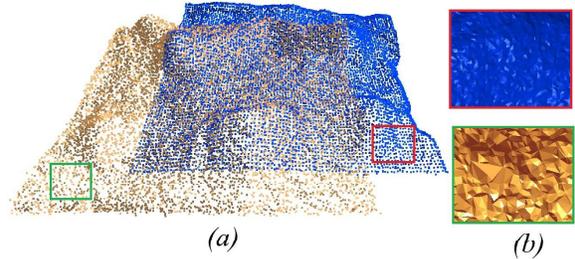


Fig. 9. (a) Result of the proposed algorithm for registering noisy point clouds with different amounts of noise; (b) the magnified sections show polygon meshes of point clouds with different noise levels.

In the next experiment, the implementation was done for the point clouds of a gilt-bronze statue (see Fig. 1(a)) scanned by a NextEngine 3D laser scanner.

Because of the surface properties of the shininess, there were noises in the point clouds. In addition, the point clouds captured by different modes of the scanner such as macro and wide range were used in the experiment. The density of the point clouds obtained by the macro-range mode was about five times higher than that of the point clouds obtained by the wide-range mode. Figure 8 shows the result of the proposed method on noisy point clouds with different densities.

In the last experiment, we registered the point clouds of a sandy terrain captured by a Kinect sensor. Both point clouds included some noise because of terrain roughness. For the experiment, we added 30% and 80% of noise to these point clouds, respectively. The registration result is shown in Fig. 9. The numerical results of our proposed algorithm are given in Table 1.

## 6 Conclusions and future work

In this paper, we presented a novel method for registering noisy point clouds. The main contributions of our research are as follows: (a) a single iteration method for eliminating noise from point clouds using FFT and low-pass filtering in the frequency domain and (b) a proposed new and robust algorithm for registering noisy point clouds.

To remove noise in point clouds, we resampled overlapped point clouds and transferred them to the frequency domain by FFT. The Fourier spectrum of the point clouds was filtered by a Gaussian function. At each stage of the method, we chose different parameters of the Gaussian function. By registering the smoothed point clouds with the ICP algorithm, we obtained the registration parameters for the original point clouds. In addition, we suggested an error measurement based on the average variation value of the registered point clouds. For noisy point clouds, this measurement was more efficient than the error measurement based on a distance.

In addition, the proposed method requires that the overlapped area of the point clouds should be an explicit discrete surface because of the DFT. This is one of the limitations of the proposed method. Another limitation of the method is that, if the number of points of the overlapped area is large, such as several millions of points, then the computation time of the method will be high because of the size of the FFT.

We experimented with our method on many different types of noisy point clouds, such as point clouds with strong noise, point clouds with different amounts of noise and noisy point clouds with different densities.

The proposed method showed robustness in the presence of strong noise. In the experiment, parameters of Gaussian window function are selected by user. The parameter values were depending on level of the noise and surface roughness.

In the future, research on automatic and optimal selection of the parameter values of the window function is required for improving application of the proposed method.

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## Шуугиант цэгэн өгөгдлийг олон алхамт шуугиан арилгах аргад тулгуурлан Фурьегийн хурдан хувиргалтаар бүртгэх нь

Н.Бямбажаргал<sup>1</sup>, Б.Анхбаяр<sup>2</sup>, Х.Оюундолгор<sup>1</sup>, А.Энхбаяр<sup>2\*</sup>

<sup>1</sup> Монгол Улсын Их Сургууль, Хэрэглээний Шинжлэх Ухаан, Инженерчлэлийн Сургууль, Мэдээлэл компьютерийн ухааны тэнхим

<sup>2</sup> Монгол Улсын Их Сургууль, Хэрэглээний Шинжлэх Ухаан, Инженерчлэлийн Сургууль, Хэрэглээний математикийн тэнхим

\*enkhbayar.a@seas.num.edu.mn

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### Хураангуй

Бид энэхүү өгүүлэлд шуугиант цэгэн өгөгдлийг бүртгэх олон алхамт аргыг танилцуулна. Алхам бүрт давхцсан дискрет гадаргуунуудыг нэгэн зэрэг Фурьегийн хурдан хувиргалт (FFT)-аар давтамжийн огторгуй руу хувиргана. Давтамжийн огторгуйд өгөгдсөн функцийг тусламжтайгаар нам давтамжуудыг шүүн авч, улмаар урвуу Фурьегийн хувиргалтаар дискрет гадаргууг дахин байгуулна. Энэхүү дахин байгуулсан гадаргууг хамгийн ойр цэгийн итераци (ICP)-ийн аргын тусламжтай бүртгэж, харгалзах бүртгэлийн параметрүүдийг тогтоосон. Эдгээр параметрүүдийг ашиглан өгөгдсөн шуугиант цэгэн өгөгдлийг бүртгэсэн. Дараагийн алхамуудыг дээрхтэй адил аргаар хэрэгжүүлсэн бөгөөд зөвхөн нам давтамжийг шүүх шүүлтүүрийн параметрүүдийг өөрчилсөн. Энэхүү процессийг цөөн алхам давтан хэрэгжүүлсний дараа бидний боловсруулсан арга шуугиант цэгэн өгөгдлийг бүртгэн авахад илүү сайн үр дүн өгсөн. Бид дээрх аргыг сийрэг цэгэн өгөгдөл, шуугианы түвшин өөр өөр байх олон төрлийн цэгэн өгөгдлийг бүртгэхэд туршиж үзсэн.

**Түлхүүр үг:** Шуугиант цэгэн өгөгдөл, Фурьегийн хурдан хувиргалт, дискрет гадаргуу

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